On Incompleteness in Abstract Argumentation: Complexity and Expressiveness

Jean-Guy Mailly

LIPADE

15th International Conference on Scalable Uncertainty Management (SUM 2022)
19 Oct. 2022
Outline

1 Background: Abstract Argumentation

2 Incomplete AFs
   - Definitions and Complexity
   - The Disjunction Problem

3 Rich Incomplete AFs

4 Constrained Incomplete AFs
   - Definition
   - Expressiveness of CIAFs
   - CIAFs and Extension Enforcement

5 Conclusion
Why Argumentation?

\( a_1 \) “I’m hungry, let’s go to this restaurant.” (John)

\( a_2 \) “The comments on Tripadvisor are bad, let’s go somewhere else.” (Yoko)

\( a_3 \) “These are old comments, and there is a new chef, so the food is probably better now.” (John)

\( a_4 \) “Moreover, all the other restaurants in this street are closed.” (John)
Why Argumentation?

\[ a_1 \] “I’m hungry, let’s go to this restaurant.” (John)
\[ a_2 \] “The comments on Tripadvisor are bad, let’s go somewhere else.” (Yoko)
\[ a_3 \] “These are old comments, and there is a new chef, so the food is probably better now.” (John)
\[ a_4 \] “Moreover, all the other restaurants in this street are closed.” (John)

Argumentation is useful when agents need to communicate about their (possibly incompatible) beliefs, goals, preferences, . . .

- strategic aspects: persuasion, negotiation, . . .

More generally, argumentation can be used to represent conflicting information and obtain reasonable outcome from it
Abstract AFs

Dung’s Argumentation Framework

**Argumentation Framework** (**AF** for short): \( F = \langle A, R \rangle \) where

- \( A \) is a set of arguments
- \( R \subseteq A \times A \) represents attacks between arguments

- Example: \( F = \langle A, R \rangle \) with
  - \( A = \{a_1, a_2, a_3, a_4\} \)
  - \( R = \{(a_2, a_1), (a_3, a_2), (a_4, a_2)\} \)

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- Collective arguments acceptability

Dung’s Semantics

Basic properties

Given $F = \langle A, R \rangle$, $S \subseteq A$ is

- conflict-free (cf) w.r.t. $F$ if $\nexists a_i, a_j \in S$ s.t. $(a_i, a_j) \in R$
- admissible (ad) w.r.t. $F$ if $S$ is cf and defends each $a_i \in S$

Classical semantics

Given $F = \langle A, R \rangle$, $S \subseteq A$ is

- complete (co) w.r.t. $F$ if $S$ is ad and contains all the arguments that it defends
- preferred (pr) w.r.t. $F$ if $S$ is a $\subseteq$-maximal co extension
- stable (st) w.r.t. $F$ if $S$ is cf and attacks every $a_j \in A \setminus E$
- grounded (gr) w.r.t. $F$ if $S$ is a $\subseteq$-minimal co extension

Semantics Example

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<tr>
<th>Semantics</th>
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Arguments Acceptability

Given $F = \langle A, R \rangle$ and $\sigma$ a semantics,

- $a \in A$ is skeptically accepted (SA) by $F$ w.r.t. $\sigma$ iff $\forall S \in \sigma(F), a \in S$
- $a \in A$ is credulously accepted (CA) by $F$ w.r.t. $\sigma$ iff $\exists S \in \sigma(F), s.t. a \in S$

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$P$-c trivial $\Pi_2^P$-c coNP-c $\Sigma_2^P$-c
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**Incomplete AFs**

**Incomplete Argumentation Framework**

**Incomplete Argumentation Framework (IAF for short):**

\[ I = \langle A, A^?, R, R^? \rangle \]

- \( A \) and \( R \) are arguments and attacks that certainly exist
- \( A^? \) and \( R^? \) are arguments and attacks that may exist, by maybe not

---

Incomplete AFs

**Incomplete Argumentation Framework**

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- \( A \) and \( R \) are arguments and attacks that certainly exist
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Why incompleteness?
- ignorance about other agents knowledge/preferences in a debate
- ignorance about the truth of arguments premises

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Completions

Completion of an IAF $I = \langle A, A^?, R, R^? \rangle$: $F = \langle A^*, R^* \rangle$ where

- $A \subseteq A^* \subseteq A \cup A^?$
- $R \cap (A^* \times A^*) \subseteq R^* \subseteq (R \cup R^?) \cap (A^* \times A^*)$

→ “classical” AF compatible with the uncertain knowledge contained in the IAF
Completions

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→ “classical” AF compatible with the uncertain knowledge contained in the IAF

Classical Reasoning with IAFs

- Possible view: the property is true in some completion
- Necessary view: the property is true in each completion

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• $a_4$ is skeptically accepted in some completion
• $a_2$ is credulously accepted in some completion

Classical Reasoning with IAFs

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- Necessary view: the property is true in each completion

- $a_3$ is skeptically accepted in each completion
- $a_4$ is skeptically accepted in some completion
- $a_2$ is credulously accepted in some completion
- $a_1$ is credulously accepted in each completion

Arguments Acceptability in IAFs

In the rest of the talk, mainly focus on:

- Possible credulous acceptability (PCA): $a$ is in some extension of some completion
- Necessary skeptical acceptability (NSA): $a$ is in each extension of each completion

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Motivation: Revising/Merging AFs

Previous work:

- **Extension-based revision of AFs**

  \[ F, \phi \xrightarrow{\text{AGM revision}} \text{Ext}_\sigma(F \ast \phi) \xrightarrow{\text{generation}} F \ast \phi = \{F'_1, \ldots, F'_k\} \]

- **Extension-based merging of AFs**

  \[ P = \langle F_1, \ldots, F_k \rangle, \mu \xrightarrow{\text{Merging under constraint}} \text{Ext}_\sigma(\Delta_\mu(P)) \xrightarrow{\text{generation}} \Delta_\mu(P) = \{F'_1, \ldots, F'_k\} \]


Example

- Suppose that the result of revising an AF yields the extensions \{a_1, a_2\}, \{a_1, a_2, a_3\}
- It is not representable with a single AF: realizability issue

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• It is representable by two AFs:

\[\begin{array}{c}
\text{a}_1 \\
\text{a}_2 \\
\text{a}_3 \\
\end{array}\]

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Paul E. Dunne, Wolfgang Dvorák, Thomas Linsbichler, Stefan Woltran: Characteristics of multiple
- It is representable by two AFs:

- Or one single IAF:

\begin{center}
\begin{tikzpicture}
  \node (a1) at (0,0) [circle,draw] {a_1};
  \node (a2) at (1,0) [circle,draw] {a_2};
  \node (a3) at (2,0) [circle,draw] {a_3};
  \draw [->] (a1) -- (a2);
  \end{tikzpicture}
\end{center}
• Suppose that the result of revising an AF yields the extensions
  \{\{a_1, a_2\}, \{a_1, a_2, a_3\}\}
• It is not representable with a single AF: realizability issue


• It is representable by two AFs:

\[\begin{array}{c}
a_1 \\
a_2 \\
a_3 \\
\end{array}\]

• Or one single IAF:

\[\begin{array}{c}
a_1 \\
a_2 \\
a_3 \\
\end{array}\]

• **Question:** Can we represent any set of AFs/extensions by a single IAF?
AF Representation by Means of IAFs

• Can we represent these AFs with one IAF?
AF Representation by Means of IAFs

Can we represent these AFs with one IAF?

Problem: this IAF has other completions

Question: Can we generalize the IAF model to represent any set of AFs/extensions?
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Rich IAFs

Main idea:

- Add a new kind of attacks, where the uncertainty concerns the direction
- Borrowed from Control AFs (Dimopoulos et al 2018)
- This new kind of uncertainty can be mixed with uncertain arguments and uncertain attacks

Yannis Dimopoulos, Jean-Guy Mailly, Pavlos Moraitis: Control Argumentation Frameworks. AAAI 2018: 4678-4685
Rich IAFs

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• Add a new kind of attacks, where the uncertainty concerns the direction
• Borrowed from Control AFs (Dimopoulos et al 2018)
• This new kind of uncertainty can be mixed with uncertain arguments and uncertain attacks

\[ a_1 \rightarrow a_2 \rightarrow a_3 \]

• There certainly is a conflict between \( a_1 \) and \( a_2 \), but we are not sure of the direction:
  • \((a_1, a_2)\),
  • \((a_2, a_1)\),
  • or both \((a_1, a_2)\) and \((a_2, a_1)\)

Yannis Dimopoulos, Jean-Guy Mailly, Pavlos Moraitis: Control Argumentation Frameworks. AAAI 2018: 4678-4685

Completions of Rich IAFs

Main result:

- The complexity of all reasoning tasks is the same, compared to IAFs
- Intuition of the proofs: guessing a completion of an IAF or guessing a completion of a RIAF is the same thing. Then verifying whether the completion satisfies some properties is also similar
Expressiveness of Rich IAFs

- Rich IAFs are strictly more expressive than IAFs

- There is no IAF with exactly these completions

- There is no (Rich) IAF with exactly these completions

Question: Can we have a framework more expressive than Rich IAFs?
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- But Rich IAFs are not “maximally” expressive

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Question

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Constrained Incomplete AFs

CIAF

\[ C = \langle A, A', R, R', \phi \rangle \] where

- \( \langle A, A', R, R' \rangle \) is a “classical” IAF
- \( \phi \) is a constraint on the completions built on \( \text{Arg}_{A \cup A'} \cup \text{Att}_{A \cup A'} \)
  - \( \text{Arg}_X = \{ \text{arg}_a \mid a \in X \} \)
  - \( \text{Att}_X = \{ \text{att}_{a,b} \mid (a, b) \in X \times X \} \)
Constrained Incomplete AFs

CIAF

\[ C = \langle A, A^?, R, R^?, \phi \rangle \] where

- \( \langle A, A^?, R, R^? \rangle \) is a “classical” IAF
- \( \phi \) is a constraint on the completions built on \( \text{Arg}_{A \cup A^?} \cup \text{Att}_{A \cup A^?} \)
  - \( \text{Arg}_X = \{ \text{arg}_a : a \in X \} \)
  - \( \text{Att}_X = \{ \text{att}_{a,b} : (a, b) \in X \times X \} \)

- Example: \( C = \langle \{a_1\}, \{a_2, a_3\}, \{(a_2, a_1), (a_3, a_1)\}, \emptyset, \text{arg}_{a_2} \oplus \text{arg}_{a_3} \rangle \)
Mapping an AF to a Formula

Given $\mathcal{A}$ the set of all possible arguments and $F = \langle A, R \rangle$ with $A \subseteq \mathcal{A}$

$$
\psi_F = (\bigwedge_{a \in A} \text{arg}_a) \land (\bigwedge_{a \in A \setminus A} \neg \text{arg}_a) \land (\bigwedge_{(a,b) \in R} \text{att}_{a,b}) \land (\bigwedge_{(a,b) \in (A \times A) \setminus R} \neg \text{att}_{a,b})
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Mapping an AF to a Formula

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- Example, with \( \mathcal{A} = \{a_1, a_2, a_3\} \):

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Mapping an AF to a Formula

Given $\mathcal{A}$ the set of all possible arguments and $F = \langle A, R \rangle$ with $A \subseteq \mathcal{A}$

$$\psi_F = (\bigwedge_{a \in A} \text{arg}_a) \land (\bigwedge_{a \in A \setminus A} \neg \text{arg}_a) \land (\bigwedge_{(a,b) \in R} \text{att}_{a,b}) \land (\bigwedge_{(a,b) \in (A \times A) \setminus R} \neg \text{att}_{a,b})$$

- Example, with $\mathcal{A} = \{a_1, a_2, a_3\}$:

  $\psi_F = \text{arg}_{a_1} \land \text{arg}_{a_2} \land \neg \text{arg}_{a_3} \land \text{att}_{a_2,a_1} \land (\bigwedge_{(a,b) \in (A \times A) \setminus R} \neg \text{att}_{a,b})$

  ![Diagram 1](image1)

- $\psi_{F'} = \text{arg}_{a_1} \land \text{arg}_{a_3} \land \neg \text{arg}_{a_2} \land \text{att}_{a_3,a_1} \land (\bigwedge_{(a,b) \in (A \times A) \setminus R} \neg \text{att}_{a,b})$

  ![Diagram 2](image2)
Mapping AFs to a CIAF

A (very) naive solution: consider all the arguments and all possible attacks as uncertain, and take the constraint $\phi = \psi_F \lor \psi_{F'}$

A (slightly) less naive solution:

$$\phi = \psi_F \lor \psi_{F'}$$

Proposition

Any set of AFs $F$ can be mapped to a CIAF s.t. its completions correspond to $F$
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Mapping AFs to a CIAF

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• A (slightly) less naive solution:

\[
\phi = \psi_F \lor \psi_{F'},
\]

\(\phi\) is a constraint that reflects the modified set of AFs.
Mapping AFs to a CIAF

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- A (slightly) less naive solution:

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**Proposition**

Any set of AFs $\mathcal{F}$ can be mapped to a CIAF s.t. its completions correspond to $\mathcal{F}$

Jean-Guy Mailly: Constrained Incomplete Argumentation Frameworks. ECSQARU 2021: 103-116
Mapping Extensions to a CIAF

• Given $\mathcal{E} = \{E_1, \ldots, E_n\}$, each $E_i \subseteq \mathcal{A}$ can be mapped to $F_i = \langle \mathcal{A}, R_i \rangle$ with $R_i = \{(a, b) \mid a \in E_i, b \in \mathcal{A} \setminus E_i\}$
• Given $\mathcal{E} = \{E_1, \ldots, E_n\}$, each $E_i \subseteq \mathcal{A}$ can be mapped to $F_i = \langle \mathcal{A}, R_i \rangle$ with $R_i = \{(a, b) \mid a \in E_i, b \in \mathcal{A} \setminus E_i\}$

• Example: $\mathcal{E} = \{\{a_1, a_2\}, \{a_2, a_3\}\}, \mathcal{A} = \{a_1, a_2, a_3\}$
Mapping Extensions to a CIAF

- Given $E = \{E_1, \ldots, E_n\}$, each $E_i \subseteq A$ can be mapped to $F_i = \langle A, R_i \rangle$ with $R_i = \{(a, b) \mid a \in E_i, b \in A \setminus E_i\}$
- Example: $E = \{\{a_1, a_2\}, \{a_2, a_3\}\}$, $A = \{a_1, a_2, a_3\}$
Mapping Extensions to a CIAF

- Given $\mathcal{E} = \{E_1, \ldots, E_n\}$, each $E_i \subseteq \mathcal{A}$ can be mapped to $F_i = \langle \mathcal{A}, R_i \rangle$ with $R_i = \{(a, b) \mid a \in E_i, b \in \mathcal{A} \setminus E_i\}$
- Example: $\mathcal{E} = \{\{a_1, a_2\}, \{a_2, a_3\}\}, \mathcal{A} = \{a_1, a_2, a_3\}$

Then, $\mathcal{F} = \{F_1, \ldots, F_n\}$ can be mapped to a CIAF (see previous result)
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- Then, $\mathcal{F} = \{F_1, \ldots, F_n\}$ can be mapped to a CIAF (see previous result)

**Proposition**

Any set of extensions $\mathcal{E}$ can be mapped to a CIAF s.t. the extensions of its completions correspond to $\mathcal{E}$
Open Question

How can we build the “best” CIAF?

- Best graph $\langle A, A^?, R, R^? \rangle$: minimize the distance w.r.t. some input
How can we build the “best” CIAF?

- Best graph \(\langle A, A^?, R, R^? \rangle\): minimize the distance w.r.t. some input

- Best formula \(\phi\) (w.r.t. size, computational property): knowledge compilation?
Complexity Issues

- Possible credulous acceptability: $a$ is in some extension of some completion
- Necessary skeptical acceptability: $a$ is in each extension of each completion
- Complexity is the same as in classical IAFs
  - Intuition: checking whether a completion satisfies the constraint is polynomial

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\sigma$-PCA</th>
<th>$\sigma$-NSA</th>
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Unpublished (but almost certain) claim:

- other acceptability problems (PSA, NCA) have the same complexity for CIAFs and standard IAFs
- verification problems are NP/coNP-c for CIAFs when they are polynomial for standard IAFs (or remain the same when they are already intractable for IAFs)
Extension Enforcement

\[ F = \langle A, R \rangle \]

\[ E \subseteq A \]

\[ \implies \]

\[ F' = \langle A', R' \rangle \]

such that \( E \) is (included in) an extension of \( F' \) for a given semantics
Extension Enforcement

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\begin{align*}
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\end{align*}
\}
\implies F' = \langle A', R' \rangle
\]

such that \( E \) is (included in) an extension of \( F' \) for a given semantics

- Normal expansion: new AF which adds new arguments and attacks, but does not change the attacks between former arguments

Ringo Baumann, Gerhard Brewka: Expanding Argumentation Frameworks: Enforcing and Monotonicity Results. COMMA 2010: 75-86
Extension Enforcement: Possibility Results

- Possibility results based on (sometimes unrealistic) examples
• Possibility results based on (sometimes unrealistic) examples

\[
\{a, d\} \text{ can be enforced as (part of) a stable extension}
\]
Possibility results based on (sometimes unrealistic) examples

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Extension Enforcement: Possibility Results

- Possibility results based on (sometimes unrealistic) examples

\{a, d\} can be enforced as (part of) a stable extension

- The existence of an “ultimate attacker” like \(x\) is not plausible in real debates
Parameterized Expansion

Given

- $F = \langle A, R \rangle$ an AF
- $\mathcal{A}$ a set of available arguments s.t. $A \cap \mathcal{A} = \emptyset$
- $\mathcal{R} \subseteq ((A \cup \mathcal{A}) \times (A \cup \mathcal{A})) \setminus (A \times A)$

we say that $F' = \langle A', R' \rangle$ is a $\mathcal{A}$-$\mathcal{R}$-parameterized expansion of $F$ iff

- $F'$ is a normal expansion of $F$,
- $A \subseteq A' \subseteq A \cup \mathcal{A}$,
- $R' = (R \cup \mathcal{R}) \cap (A' \times A')$. 
Parameterized Expansion

Given

- \( F = \langle A, R \rangle \) an AF
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we say that \( F' = \langle A', R' \rangle \) is a \( A-R \)-parameterized expansion of \( F \) iff

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- \( A \subseteq A' \subseteq A \cup A \),
- \( R' = (R \cup R) \cap (A' \times A') \).

\( A \) and \( R \) encode the possible actions of the agent in the debate.
Parameterized Enforcement

From $F = \langle A, R \rangle$, $A$ and $R$

- we call “possible action” any $F'$ that is a $A$-$R$-parameterized expansion of $F$
- we can build a CIAF $C$ s.t. its completions correspond to the possible actions
Parameterized Enforcement

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**Proposition**

The set of arguments $S$ can be enforced in $F$ iff it is credulously accepted w.r.t. some completion of $C$
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**Proposition**

The set of arguments $S$ can be enforced in $F$ iff it is credulously accepted w.r.t. some completion of $C$

- Research on CIAFs provides means for implementing realistic methods for enforcing extensions in argument-base dialogue
1. Background: Abstract Argumentation

2. Incomplete AFs
   - Definitions and Complexity
   - The Disjunction Problem

3. Rich Incomplete AFs

4. Constrained Incomplete AFs
   - Definition
   - Expressiveness of CIAFs
   - CIAFs and Extension Enforcement

5. Conclusion
Related Work: Constraining the completions

- Independently of our work, Herzig and Yuste-Ginel defined a framework equivalent to our CIAFs (using the same name), and proved that it is “maximally” expressive
- Fazzinga et al. studied Argument-Incomplete AFs (IAFs with $R^? = \emptyset$) and Attack-Incomplete AFs (IAFs with $A^? = \emptyset$) with correlations, which are special cases of our constraints ($X \rightarrow Y$, $X \lor Y$, $\neg(X \land Y)$, $X \oplus Y$), and focus on the possible verification problem
  - Given an IAF with correlations $I$ and a set of arguments $S$, is $S$ an extension of some completion of $I$?

Andreas Herzig, Antonio Yuste-Ginel: Abstract Argumentation with Qualitative Uncertainty: An Analysis in Dynamic Logic. CLAR 2021: 190-208
Bettina Fazzinga, Sergio Flesca, Filippo Furfaro: Reasoning over Argument-Incomplete AAFs in the Presence of Correlations. IJCAI 2021: 189-195
Related Work: IAFs without completions

- Semantics have been defined for reasoning with Partial AFs (\(\simeq\) Attack-Incomplete AFs, i.e. IAFs with \(A = \emptyset\)) without using the set of completions \(\text{(Cayrol et al 2007)}\), by adapting the definitions of conflict-freeness and defense to this setting

- Recent work: generalization to the work by \(\text{(Cayrol et al 2007)}\) to IAFs, with more semantics studied \(\text{(Mailly 2021, Mailly 2023)}\)
  - Same complexity as in Dung’s framework
  - Available SAT-based solver

Claudette Cayrol, Caroline Devred, Marie-Christine Lagasquie-Schiex: Handling Ignorance in Argumentation: Semantics of Partial Argumentation Frameworks. ECSQARU 2007: 259-270
Jean-Guy Mailly: Extension-Based Semantics for Incomplete Argumentation Frameworks. CLAR 2021: 322-341
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Summary

• CIAFs increase the expressivity of IAFs without increasing the complexity for
  • possible credulous acceptability
  • necessary skeptical acceptability

• Suitable representation of “disjunction” of AFs or extensions
  • useful for AF revision or AF merging

• Encodes a new type of enforcement operator
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Future work

- Design methods for choosing the optimal CIAF corresponding to a set of AFs/extensions
- Implement AF revision/merging based of CIAFs
- Implement extension enforcement based on CIAFs
- Define negotiation methods based on CIAFs
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More details:
Jean-Guy Mailly: On Incompleteness in Abstract Argumentation: Complexity and Expressiveness. SUM 2022: 19–33