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Non-specificity-based Supervised Discretization for Possibilistic Classification

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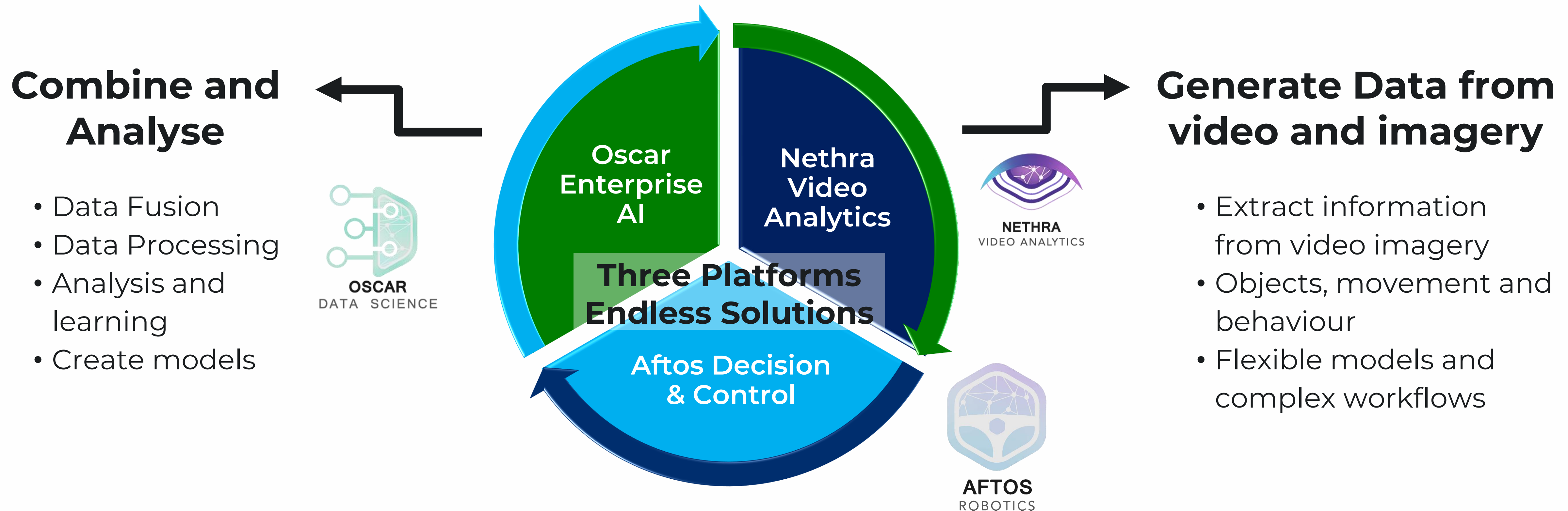
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Context and Motivation

MAL - Decision Intelligence

Massive Analytic is a Deep-tech Scale-up specialising in Decision Intelligence.

MAL's suite of platforms can be integrated for implementation to generate world-beating decision support and control outputs.



Automate, Simulate, Predict, Decide and Control

- Complex scenarios
- Many actors and variable
- Simulate and predict scenarios
- Real-time operations
- Strategic option analysis

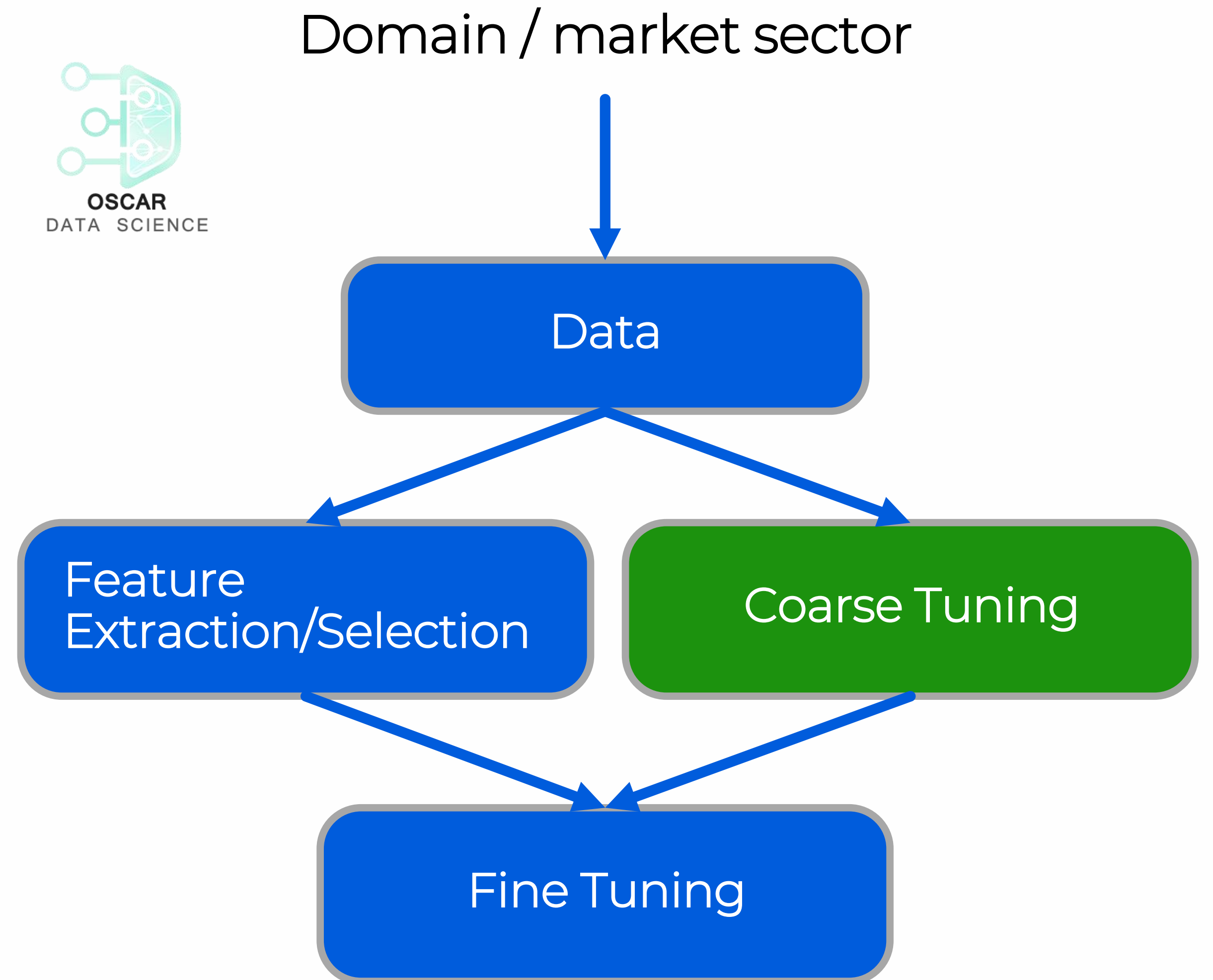


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OSCAR Workflow

- Coarse Tuning: deploy **possibilistic algorithms** not available in competitive platforms
- Feature Extraction/Selection: extend the performance and explainability of in-house solutions with new datasets
- Fine Tuning: Outperform competitive platforms in the achieved accuracies of AI pipelines



Problem Statement & Objective

- Real Data: multiple formats (numerical, categorical, mixed)
- Possibilistic Decision Trees [Jenhani et al. 2008] ONLY handle Categorical features.
- Discretization of numeric features requires ignoring the possibilistic labels!

How to discretize numeric features without ignoring the uncertainty in the class labels?

Possibilistic Datasets and Non-Specificity based Possibilistic Decision Trees (NSPDT)

Jenhani I., Ben Amor N., Elouedi Z. :**Decision Trees as Possibilistic Classifiers**, *International Journal of Approximate Reasoning*, 48(3):784–807, 2008.

Standard Dataset Vs. Possibilistic Dataset

n instances; m features; c class labels

Standard dataset

	Feature 1	Feature 2	...	Feature m	Class label
1	23.5	0	...	Low	C_2
2	13.75	1	...	Average	C_1
...
n	20	1	...	High	C_1

Ground truth: each instance belongs to only one category and we know it with certainty

Possibilistic dataset

	Feature 1	Feature 2	...	Feature m	C_1	C_2	...	C_c
1	23.5	0	...	Low	0	14
2	13.75	1	...	Average	1	0	...	0
...
n	20	1	...	High	1	1	...	0

Ground truth: each instance belongs to only one category but we are uncertain about it

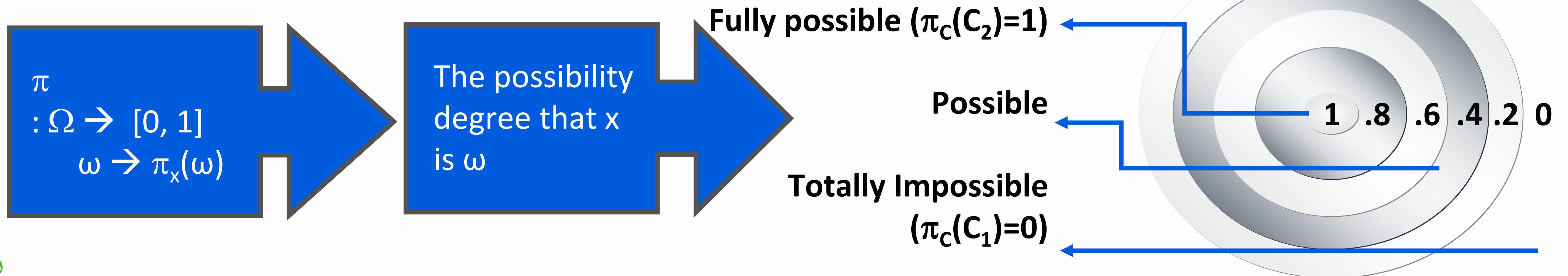
Possibilistic Dataset

Notations π

- Ω : the universe of discourse (e.g. $\{C_1, \dots, C_c\}$)
- x : a variable with an unknown value (e.g. The class label of an object)
- ω : an element of Ω (e.g. C_2)
- L : the possibilistic scale (e.g. $[0, 1]$)

	Feature 1	Feature 2	...	Feature m	C_1	C_2	...	C_c
1	23.5	0	...	Low	0	14
2	13.75	1	...	Average	1	0	...	0
...
n	20	1	...	High	1	1	...	0

Possibility distribution π



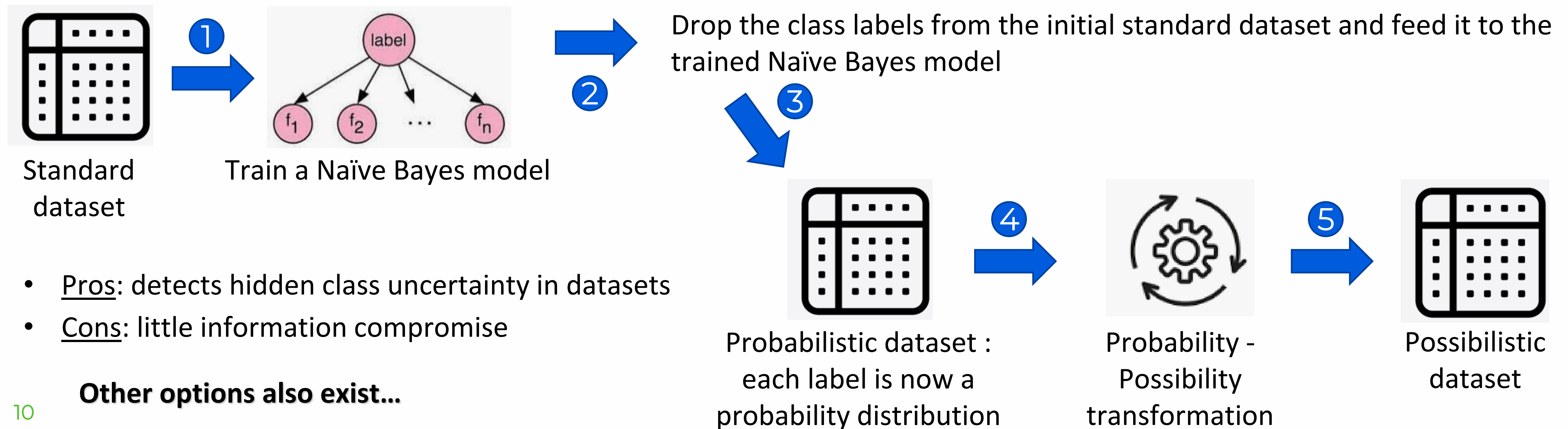
Possibilistic dataset – how to get it?

Option 1: a SME will annotate the dataset with the possibility distributions

[Challenge: SME should understand that a possibility distribution is not a probability distribution...]

Option 2: Generate the possibilistic dataset from a standard dataset using the following procedure:

	Feature 1	Feature 2	...	Feature m	C ₁	C ₂	...	C _c
1	23.5	0	...	Low	0	14
2	13.75	1	...	Average	1	0	...	0
...
n	20	1	...	High	1	1	...	0



- Pros: detects hidden class uncertainty in datasets
- Cons: little information compromise

Other options also exist...

Building decision trees from training sets with imprecise class labels using the concept of non-specificity

Imprecise Tr

Income	Property	UnCredit	C ₁	C ₂	C ₃
High	Greater	No	1	0.1	0.3
High	Greater	Yes	0.8	1	0.6
High	Greater	No	1	0.5	0.3
High	Less	Yes	0	1	0
Average	Greater	No	1	0	0.4
Average	Greater	Yes	0.7	1	0.2
Average	Less	No	0.7	1	0.7
Average	Less	Yes	0	1	0.3
Low	Less	No	0.5	0.5	1
Low	Less	Yes	0	0.3	1

NS-PDT: attribute selection measure

Using Non-Specificity

Partition 1

[1 0.3 0.6] [0 1 0.7]
[1 0 0] [1 0.3 1]
[0 0.6 1] [0.5 1 0.1]

$$\rightarrow \pi_{\text{avg}}^1 [0.58 \ 0.53 \ 0.56]$$

$$\rightarrow \pi_{\text{rep}}^1 [1 \ 0.91 \ 0.96]$$

$$\rightarrow U(\pi_{\text{rep}}^1) = 1.49$$

Partition 2

[1 1 0.3] [0.7 0 1]
[0.2 0.8 1] [1 0.3 1]
[1 0.6 0] [0 1 0]

$$\rightarrow \pi_{\text{avg}}^2 [0.65 \ 0.61 \ 0.55]$$

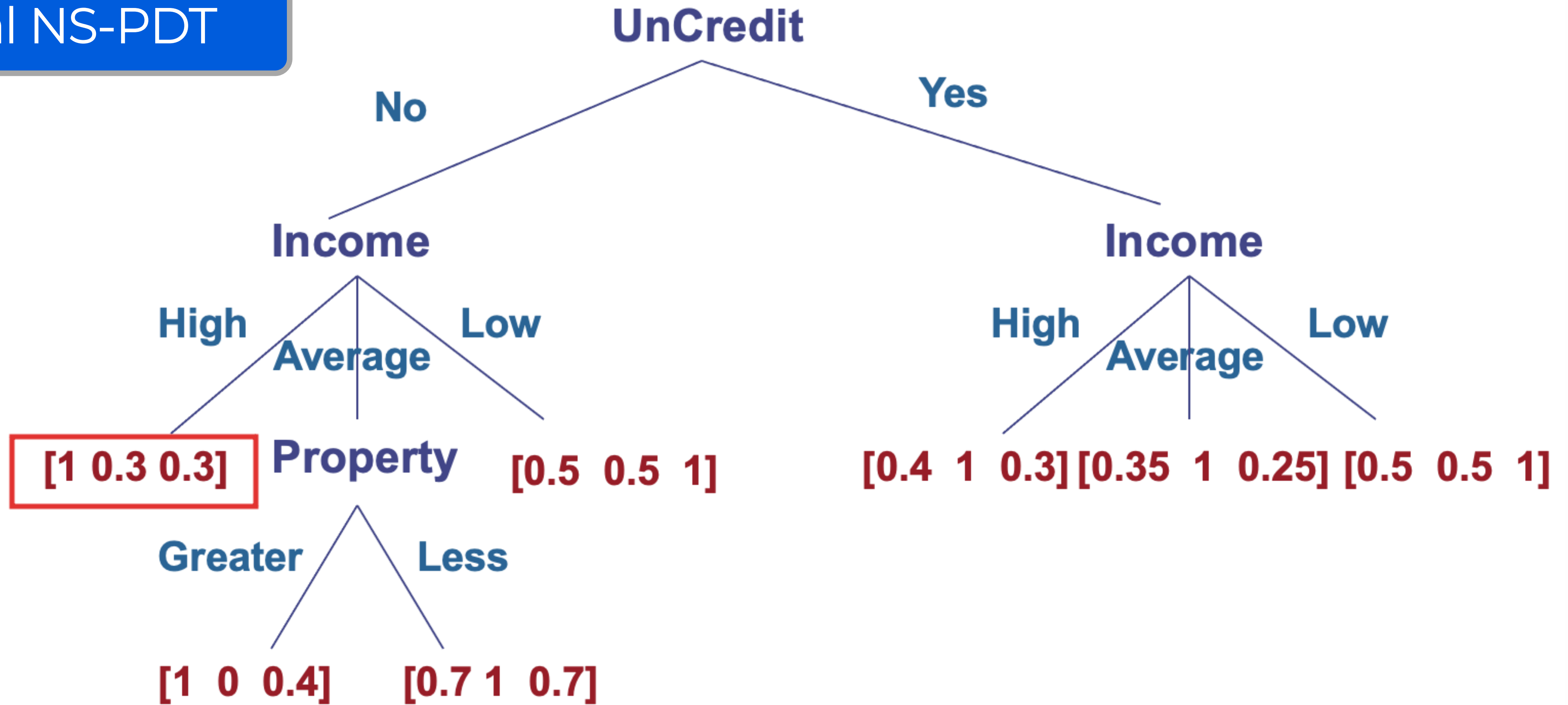
$$\rightarrow \pi_{\text{rep}}^2 [1 \ 0.93 \ 0.84]$$

$$\rightarrow U(\pi_{\text{rep}}^2) = 1.42$$

→ Partition n°2 is more specific

Classification using NS-PDT

→ Final NS-PDT



We can predict the class of the instance: <High, Greater, No>:?



The Non-Specificity based Discretization

Standard Discretization Algorithm [Fayyad and Irani, 93]

	F_i	L
	0.1	O
	0.1	O
	0.1	O
t_1	0.3	X
	0.3	X
	0.4	X
	0.5	X
t_2	0.5	X
	0.5	O
	0.6	O
	0.6	O
	0.7	O
	0.7	O
t_3	0.8	O
	0.9	X
t_4	0.9	O
	0.9	O
t_5	0.9	O
	0.9	X
	0.9	X

- Sort instances in ASC order (by F_i)
- Find cut point t that **maximizes $\text{Gain}(T,t)$**
 t : the average between 2 F_i values around the boundary point (when class label changes).
 T : training partition, t : threshold that will split T into 2 sub-partitions:
 - T_1 : Instances where $F_i \leq t$
 - T_2 : Instances where $F_i > t$
- Repeat from 2 for each generated sub-partition until stopping criterion is met

15 Total: 19 instance, 8: X 11: O



Possibilistic Discretization Algorithm

	F_i	$\pi(X)$	$\pi(O)$
	0.1	1	0.8
	0.1	0.5	1
	0.1	0.8	1
t_1	0.3	1	0.2
	0.3	1	0.6
	0.4	1	0
t_2	0.5	0.1	1
t_3	0.5	1	0.5
	0.5	1	0.3
t_4	0.6	0	1
	0.6	0.2	1
t_5	0.7	1	0
	0.7	1	0.6
	0.8	0.8	1
	0.9	1	0.7
t_6	0.9	0.3	1
t_7	0.9	1	0.6
	0.9	1	0.2
	0.9	1	0.1

16 Total: 19 instance

1. Sort instances in ASC order (by F_i)

2. Find cut point t that maximizes $NSGain(T,t)$

Cut points: when $InfoAff(\pi_i, \pi_{i+1}) \leq \eta$ (hyper-parameter, default: 0.7)

T: training partition

t: cut point that will split T into 2 sub-partitions:

- T_1 : Instances where $F_i \leq t$
- T_2 : Instances where $F_i > t$

3. Repeat from 2 for each generated sub-partition until stopping criterion is met: $NSGain \leq 0$

$$InfoAff([0.8,1], [1,0.2]) = 1 - \frac{\left(\frac{0.2 + 0.8}{2}\right) + 1 - (\max(\min(0.8,1), \min(1,0.2)))}{2}$$

$$= 1 - \frac{0.5+0.2}{2} = 0.65 (\leq 0.7 \text{ so cut point})$$

For t_1 : $NSGain(T, t_1 = 0.2) = U(\pi_{rep}(T)) - (freq_{T_1} * U(\pi_{rep}(T_1)) + freq_{T_2} * U(\pi_{rep}(T_2)))$

$\pi_{rep}(T)$: max – normalized average possibility distribution representing T

Possibilistic Similarity Measure

Information Affinity: [Jenhani, Benferhat and Elouedi; ECSQARU'07, IPMU'08, Foundation of Reasoning under Uncertainty, 2010]

Definition let π_1 and π_2 be two possibility distributions on the same universe of discourse Ω . We define a measure $InfoAff(\pi_1, \pi_2)$ as follows:

$$InfoAff(\pi_1, \pi_2) = 1 - \frac{d(\pi_1, \pi_2) + Inc(\pi_1 \wedge \pi_2)}{2}$$

Where $d(\pi_1, \pi_2) = \frac{1}{n} \sum_{i=1}^n |\pi_1(\omega_i) - \pi_2(\omega_i)|$ represents the Manhattan distance between π_1 and π_2 and $Inc(\pi_1 \wedge \pi_2)$ tells us about the degree of conflict between the two distributions.

$$Inc(\pi) = 1 - \max_{\omega \in \Omega} \{\pi(\omega)\}$$

Non-specificity Measure

U-uncertainty: *[Klir and Folger, 1988]*

$$U: \mathcal{R} \rightarrow \mathbb{R}^+$$

where \mathcal{R} denotes the set of all finite, ordered, and normal possibility distributions.

Given a possibility distribution

$$r = (r_1, r_2, \dots, r_n) \text{ such that } 1 = r_1 \geq r_2 \geq \dots \geq r_n$$

The U-uncertainty of r , $U(r)$, can be expressed by :

$$U(r) = \sum_{i=2}^n (r_i - r_{i+1}) \log_2 i = \sum_{i=2}^n r_i \log_2 \frac{i}{i-1}$$

where $r_{n+1} = 0$ by convention.

Algorithm: Illustration

	F_i	$\pi(X)$	$\pi(O)$
	0.1	1	0.8
	0.1	0.5	1
	0.1	0.8	1
t_1	0.3	1	0.2
	0.3	1	0.6
t_2	0.4	1	0
t_3	0.5	0.1	1
t_4	0.5	1	0.5
	0.5	1	0.3
t_5	0.6	0	1
	0.6	0.2	1
t_6	0.7	1	0
	0.7	1	0.6
	0.8	0.8	1
t_7	0.9	1	0.7
	0.9	0.3	1
	0.9	1	0.6
	0.9	1	0.2
	0.9	1	0.1

19 Total: 19 instance

$$\pi_{T_1}(X) = (1+0.5+0.8+1+\dots+1)/19 = 0.774$$

$$\pi_{T_1}(O) = (0.8+1+1+0.2+\dots+0.1)/19 = 0.61$$

$\pi_{rep(T_1)}(X)$	$\pi_{rep(T_1)}(O)$
1	0.79

$$U(\pi_{rep}(T_1)) = (1-0.79) * \log_2(1) + (0.79-0) * \log_2(2) = 0.79$$

$$\pi_{T_2}(X) = (1+1+1+0.1+\dots+1)/16 = 0.775$$

$$\pi_{T_2}(O) = (1+0.6+0+1+\dots+0.1)/16 = 0.55$$

$\pi_{rep(T_2)}(X)$	$\pi_{rep(T_2)}(O)$
1	0.7

$$U(\pi_{rep}(T_2)) = (1-0.7) * \log_2(1) + (0.7-0) * \log_2(2) = 0.7$$

$$\pi_{T_1}(X) = (1+0.5+0.8)/3 = 0.766$$

$$\pi_{T_1}(O) = (0.8+1+1)/3 = 0.933$$

$\pi_{rep(T_1)}(X)$	$\pi_{rep(T_1)}(O)$
0.821	1

$$U(\pi_{rep}(T_1)) = (1-0.821) * \log_2(1) + (0.821-0) * \log_2(2) = 0.821$$

$$NSGain(T, t_1=0.2) = 0.79 - [(16/19) * 0.7 + (3/19) * 0.821] = 0.07$$

Similarly, we compute $NSGain(T, t_2=0.45)$, $NSGain(T, t_3=0.5)$, ... then select max

The consecutive selected thresholds will form the cutoff points.



Experimental Setup and Results

Datasets [UCI ML Repository]

Dataset	#instances	#classes	#features	#continuous features
Letter recognition	20 000	26	16	16
Dry Bean	13 611	7	16	16
Magic Gamma Telescope	19 020	2	10	10
Occupancy Detection	9 752	2	5	5
Spambase	4 601	2	57	57
Adult	48 842	2	14	6
Bank Marketing	45 211	2	20	10

Possibilistic versions of these datasets have been generated using Option 2 (NB + Proba-Poss transformation)



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Results – Crisp datasets

Dataset	Classifier	Mean Acc. (Non dic.)	Mean Acc. (Std. disc.)	Mean Acc. (NS-disc.)
Letter Recognition	NB	64.01	73.85	70.6
	J48	87.92	78.75	80.59
	RF	96.41	93.41	92.61
	1-NN	95.96	91.86	89
	LogReg	91.24	92.84	90.14
Dry Bean	NB	89.7	89.94	88.5
	J48	91.03	90.05	90.25
	RF	92.5	91.47	91.14
	1-NN	90.2	89.25	90.2
	LogReg	92.6	92.3	92.6
Magic Gamma Telescope	NB	72.68	78.25	75
	J48	85.05	84.45	77.57
	RF	88	84.04	77.46
	1-NN	80.93	81.97	77.42
	LogReg	79.11	84.68	77.5
Occupancy Detection	NB	95.34	99.1	99.33
	J48	95.86	99.37	99.37
	RF	99.4	99.4	99.37
	1-NN	94.99	99.37	99.37
	LogReg	99.24	99.35	99.35
Spambase	NB	85.24	90.19	85.78
	J48	82.59	92.82	91.58
	RF	95.5	94.59	93.87
	1-NN	85.28	93.1	92.24
	LogReg	92.41	94.41	91.58
Adult	NB	83.25	83.87	82.35
	J48	86.1	86.67	85.45
	RF	85.17	85.4	84.75
	1-NN	79.52	83.04	84.13
	LogReg	85.09	87.23	85.58
Bank Marketing	NB	88	88.88	88.88
	J48	90.31	90.32	88.88
	RF	90.38	89.92	88.87
	1-NN	86.96	88.83	88.87
	LogReg	90.15	90.4	88.87

Crisp dataset

F1	F 2	...	F m	C ₁	C ₂	...	C _c
23.5	0	...	Low	0	1	0	0
13.75	1	...	Average	1	0	0	0
...
20	1	...	High	0	1	0	0



Results - Possibilistic datasets

Dataset (Discretized)	NS-PDT
Letter recognition	<ul style="list-style-type: none"> • CMPcc: 60.2% • InfoAffC: 0.871
Dry Bean	<ul style="list-style-type: none"> • CMPcc: 88.47% • InfoAffC: 0.963
Magic Gamma Telescope	<ul style="list-style-type: none"> • CMPcc: 81.65% • InfoAffC: 0.836
Occupancy Detection	<ul style="list-style-type: none"> • CMPcc: 95.9% • InfoAffC: 0.956
Spambase	<ul style="list-style-type: none"> • CMPcc: 97.9% • InfoAffC: 0.84
Adult	<ul style="list-style-type: none"> • CMPcc: 96.66% • InfoAffC: 0.96
Bank Marketing	<ul style="list-style-type: none"> • CMPcc: 86.82% • InfoAffC: 0.882

CMPcc: Cautious Most Plausible-based correct classification

$$CMPcc = \frac{\text{number of correctly classified instances}}{\text{total number of testing instances}} \times 100$$

InfoAffC: Information Affinity Criterion

$$InfoAffC = \frac{1}{n} \sum_{i=1}^n InfoAff(\pi_i^{init}, \pi_i^{pred})$$



Conclusion and Future work

Conclusion and Future work

- A supervised discretization approach has been proposed for possibilistic (labelled) data
- Non-specificity and Information Affinity measures are used by the main building blocks of the proposed algorithm.
- The proposed pre-processing discretization approach will make it possible to use several possibilistic classifiers which were initially designed to handle categorical data only.
- With crisp data, NS discretization showed competitive results.
- More suitable for possibilistic data

Future work:

- Consider feature dependency during discretization instead of discretizing each feature individually.
- Handle unbalanced feature intervals.



Thank you!



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