Identifying and repairing inconsistencies in preference elicitation

SUM 2022

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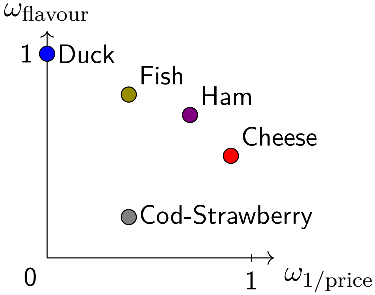
Plan

1. Context: why repairing and how?
2. Maximal Coherent Subsets: an interesting solution
3. MCS and preference elicitation

Example: multiple-criteria decision

- Questions: how can I find my favourite pizza?

	Flavour	1/price
Cheese	5	9
Duck	10	0
Fish	8	4
Ham	7	7
Cod-strawberry	2	4



- Supposition: agent's preferences = aggregation function.
- Here: $f_\omega(\text{pizza}) = 0.6\ \text{flavour} + 0.4\ 1/\text{price}$.

Example: multiple-criteria decision

- Questions: how can I find my favourite pizza?

	Flavour	1/price	f_ω
Cheese	5	9	6.6
Duck	10	0	6
Fish	8	4	6.4
Ham	7	7	7
~~Cod strawberry~~	~~2~~	~~4~~	~~2.8~~

- Supposition: agent's preferences = aggregation function.
- Here: $f_\omega(\text{pizza}) = 0.6 \text{ flavour} + 0.4 \ 1/\text{price}$.
- Best: ham with a score of 7. Cod-strawberry always dominated.

Why elicitation?

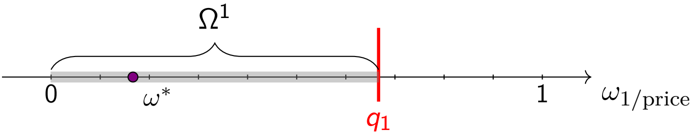
Problem: in practice, the parameters ω are unknown.

- An expert chooses a parametric family of aggregate functions of criteria f_ω (weighted sum, OWA...) describing the preferences.
- The expert searches $\omega \in \Omega$ through an elicitation[1] of the user's preferences with explicit questions (pairwise comparison).
- Here: incremental [1] robust [2, 3] elicitation $\Rightarrow$ strong performance guarantees, supposing no errors in the answers (oracle) and in the choice of f_ω.

[1] Elicitation: collect and formalize human knowledge for further exploitation.

Find ω^*

- The agent answers correctly three questions q_1, q_2 and q_3 by comparing each time two alternatives.
- Each answer refines the set of possible models such that $\omega^* \in \Omega^3 \subset \Omega^2 \subset \Omega^1 \subset \Omega$.



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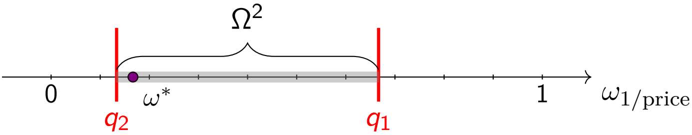

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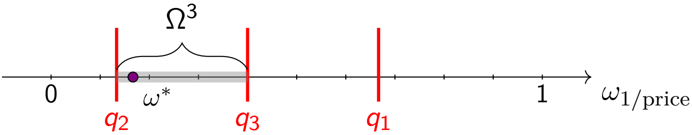
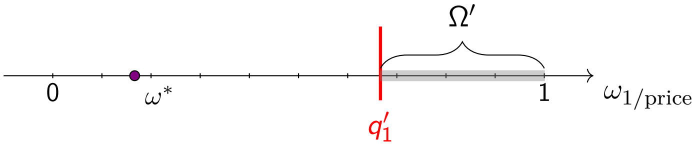


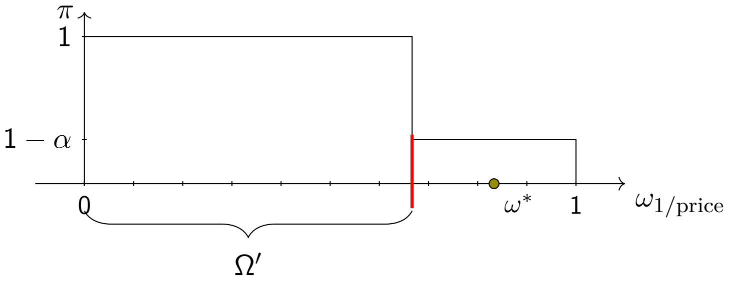
What happens in case of error?

- Let us suppose the agent gives a wrong answer to the question q'_1, then the optimal model ω^* is not part of Ω':



- Further questions will refine Ω', thus never returning to the optimal model.

A solution: possibilistic elicitation [4]



- The agent gives a confidence level $\alpha \in [0, 1]$ with each answer.
- Robust to wrong answers/models (possible to return to $\Omega \setminus \Omega'$), and we can detect inconsistency easily.

Limitations: why we want to repair

- As inconsistency increases, recommendations are likely to become less optimal, but nothing is done to handle inconsistency.

  ⇒ instead of stopping early, we could remove/correct inconsistency to continue elicitation and improve the final recommendation.

- While we can detect both sources of inconsistency (user and model), we have no information on the source of the inconsistency.

  ⇒ determining the source of inconsistency is important to correct it, but also to improve the elicitation process.

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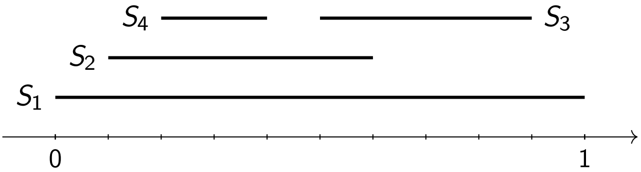
Maximal Coherent Subsets (MCS)

- Definition of MCS: groups of consistent sources that are as big as possible.
- Two answers q_1 and q_2 are consistent: the intersection of the subsets of possible models Ω^1 and Ω^2 is non-empty (represented with polytopes).
- A more formal definition: set of sets $\mathcal{S} = \{S_1, ..., S_n\}$ with:
  - Non-empty intersection: $\bigcap_{i=1}^{n} S_i \neq \emptyset$.
  - Maximal intersection: $\forall P \notin \mathcal{S}, \bigcap_{i=1}^{n} S_i \cap P = \emptyset$.

Example : MCS

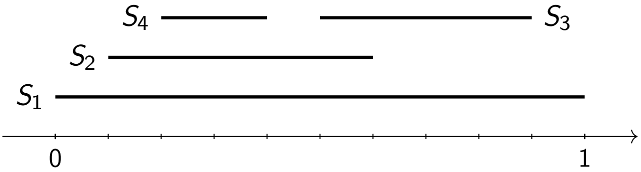
What are the MCSs?



Answer:

Example : MCS

What are the MCSs?



Answer: $\{1, 2, 3\}$ and $\{1, 2, 4\}$.

Why and why not MCS?

Pros:

- A natural way of coping with inconsistent information: separate inconsistent sources into multiple consistent groups.
- Require no additional information on the sources.
- MCSs have been used in the past both in logic [5] and in numerical settings [6].

Con:

- Enumerating MCSs is a NP-hard problem in general (2^N subsets).

A solution: approximation with hyperrectangles

- Enumerating MCS is computationally easy (polynomial) with intervals [7].
- We proved that this can be used to efficiently find MCS for axis-aligned hyperrectangles [8].
- Possible to approximate polytopes with inner and outer approximations based on such hyperrectangles.

  $\Rightarrow$ we have approximations of the different MCSs.

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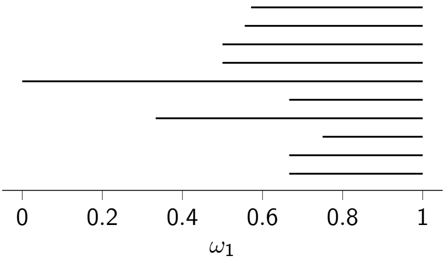
Normal elicitation

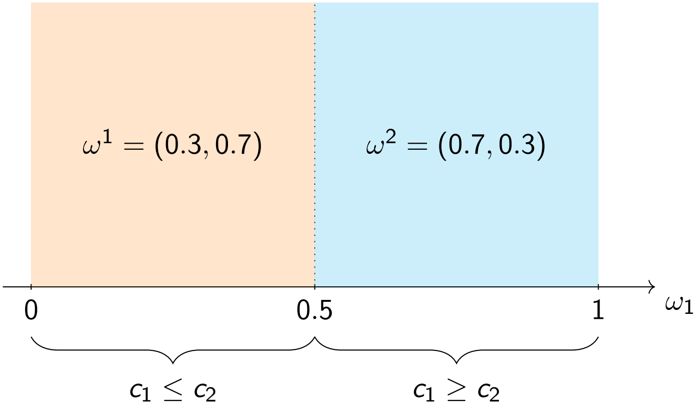


- All answers have at least one common $\omega \in \Omega$.

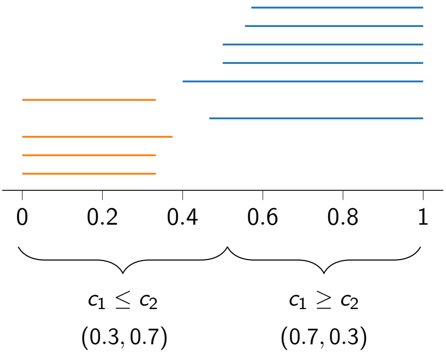
  $\Rightarrow$ no inconsistency, giving an optimal recommendation.

Elicitation with an OWA model

- An OWA can be seen as a piecewise weighted sum. Supposing we have an OWA with $\omega^* = (0.7, 0.3)$:



Elicitation with a wrong model (OWA right, WS supposed)

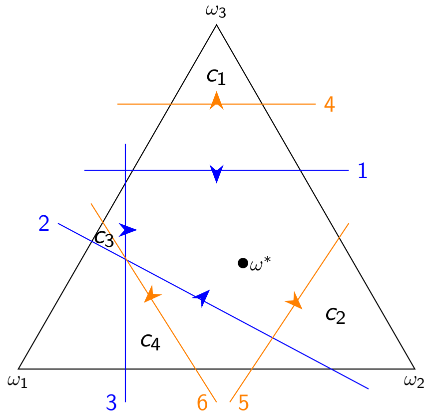


- Two MCSs that are roughly the same size and dissociated.

  $\Rightarrow$ easy to conclude there is a model error.

- We can switch to a more expressive (k-Choquet) or alternate (OWA) model.

Elicitation with wrong answers (3 correct, 3 incorrect)



- MCSs:
  - $c_1 = \{2, 3, 4\}$,
  - $c_2 = \{1, 2, 3, 5\}$,
  - $c_3 = \{1, 2, 6\}$,
  - $c_4 = \{1, 3, 6\}$.
- MCSs are roughly the same size, with many overlaps.

  $\Rightarrow$ hard to determine and thus remove/correct errors.

What to do now

- We can differentiate model (systematic) and user (random) errors. However, identifying user errors finely seems difficult.

  $\Rightarrow$ probably due to the relatively poor information given by each answer (quite large sets).

- We plan to do larger experiments (more dimensions, optimal questions...).

- For user errors, we also want to consider different solutions:
  - Use confidence levels to determine which MCS to keep, and therefore remove/correct errors.
  - Instead of pairwise comparison, the user picks her favourite alternative between three.
  - ...

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