WWU
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A Glimpse into Statistical Relational AI

The Power of Indistinguishability

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Agenda

- Statistical Relational Artificial Intelligence
  - Probabilistic relational models
  - Grounding semantics
  - Context
- The Power of Indistinguishability
  - Lifted query answering and tractability
  - Keeping indistinguishability over time
  - Indistinguishability in decision making
- Summary

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Statistical Relational Artificial Intelligence (StaRAI)

AI: intelligent systems
in the real world

Figure based on Stuart Russell

Statistical Relational Artificial Intelligence (StaRAI)

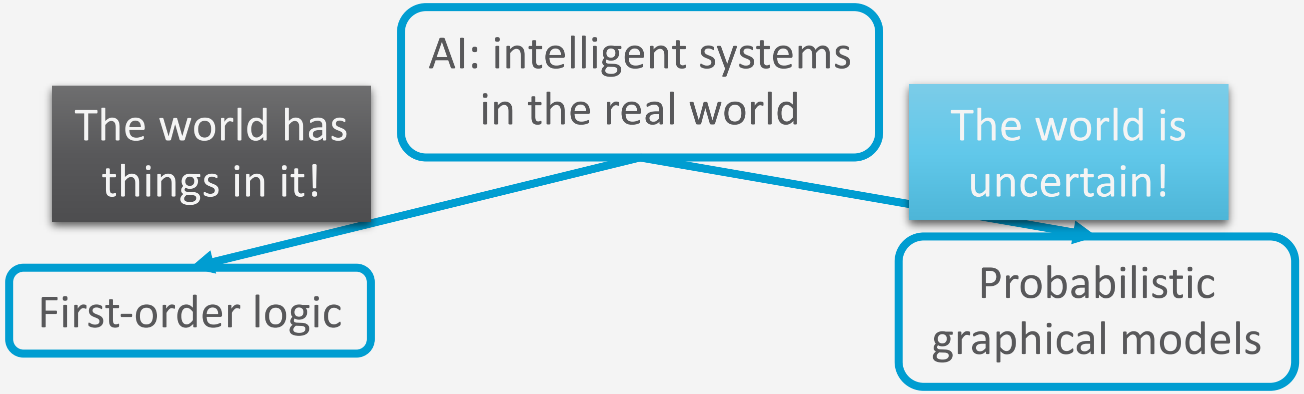
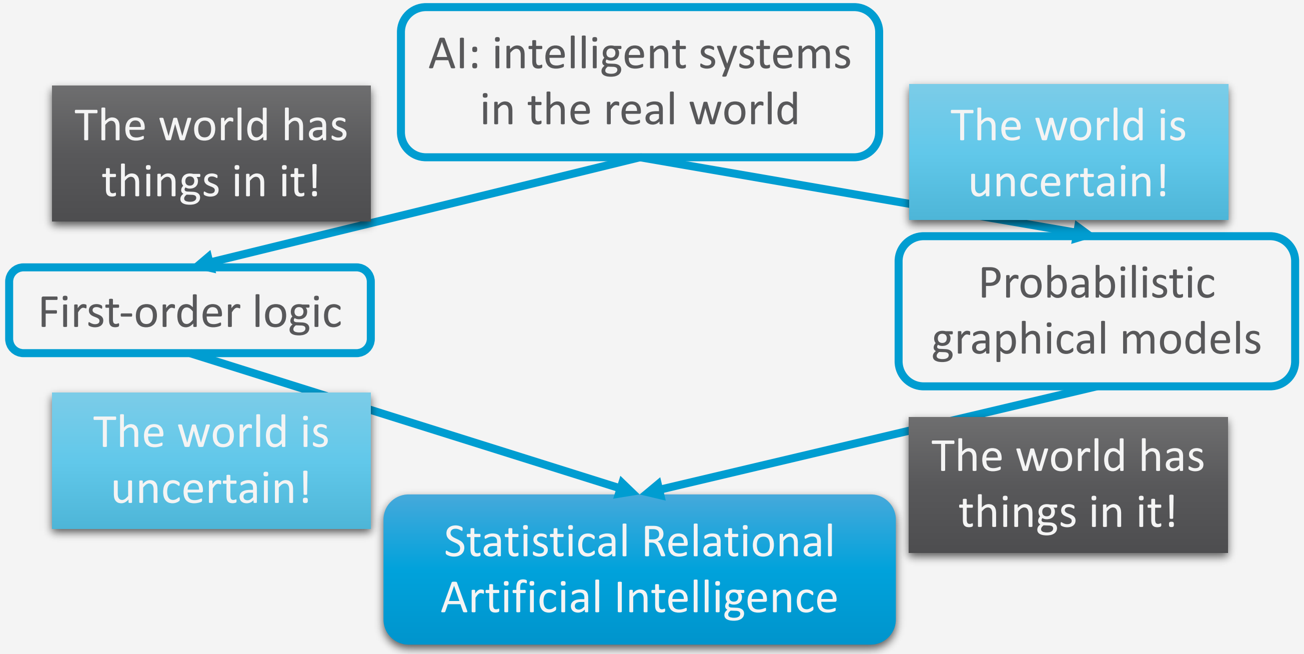


Figure based on Stuart Russell

Statistical Relational Artificial Intelligence (StaRAI)



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Statistical Relational Artificial Intelligence (StaRAI)

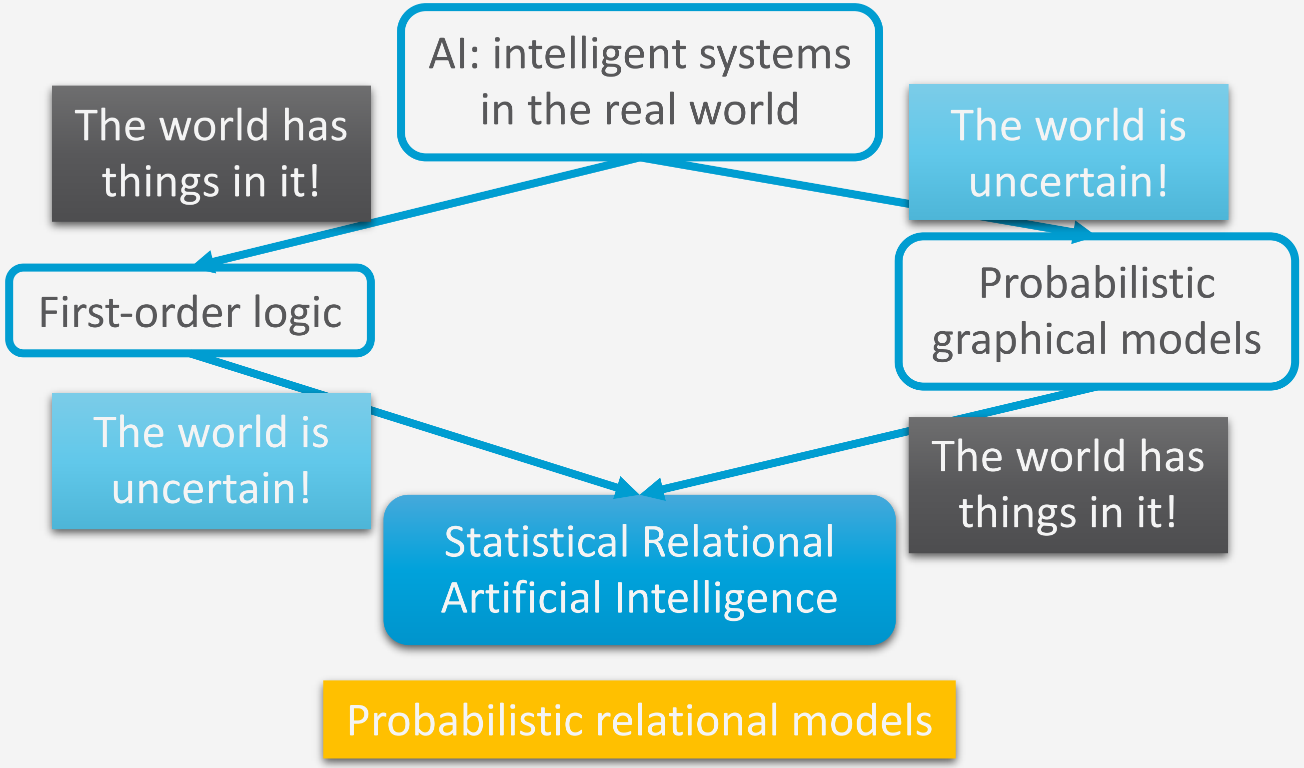
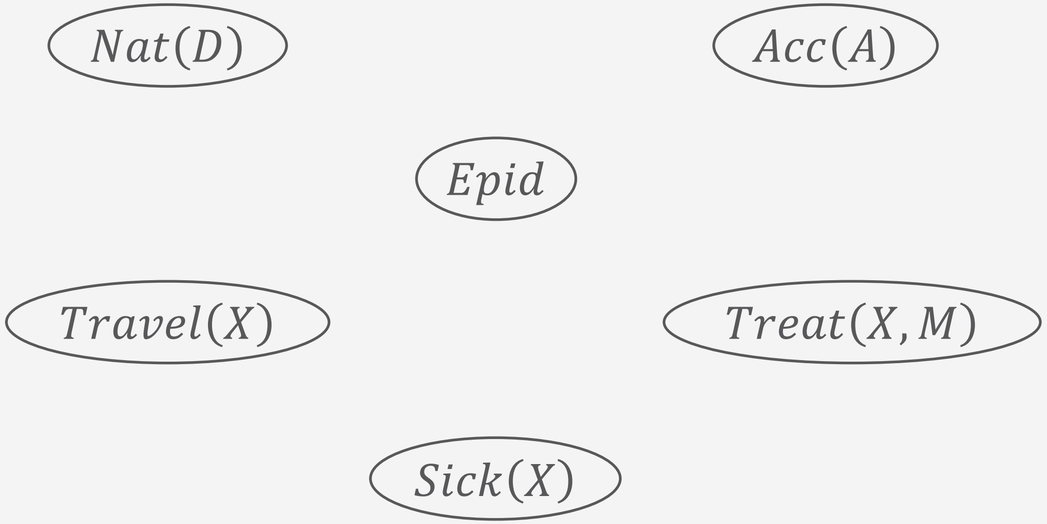


Figure based on Stuart Russell

Application: Epidemics

- Atoms: Parameterised random variables = PRVs
  - With logical variables
    - E.g., X, M
    - Possible values (domain):
      $$dom(X) = \{alice, eve, bob\}$$
      $$dom(M) = \{injection, tablet\}$$
  - With range
    - E.g., Boolean
    - $ran\big(Travel(X)\big) = \{true, false\}$
  - Represent sets of *indistinguishable* random variables

$$Nat(D) = natural\ disaster\ D$$
$$Acc(A) = accident\ A$$



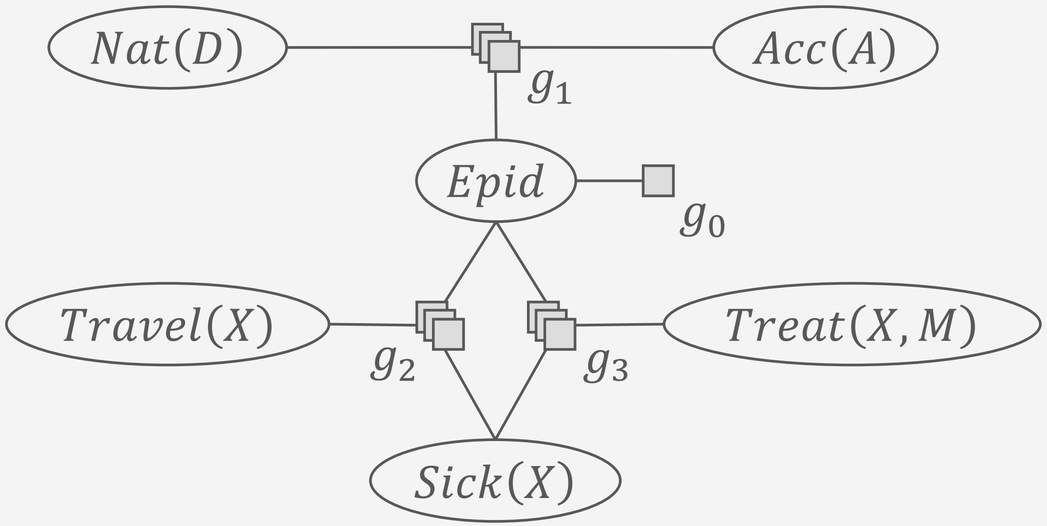
Encoding the Joint Distribution: Factorisation

- Factors with PRVs = parfactors
  - E.g., g_2

$Travel(X)$	$Epid$	$Sick(X)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

Potentials

- In parfactors, just like in factors, no probability distribution as factors required



Factors

- Grounding
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

$Travel(X)$	$Epid$	$Sick(X)$	g_2
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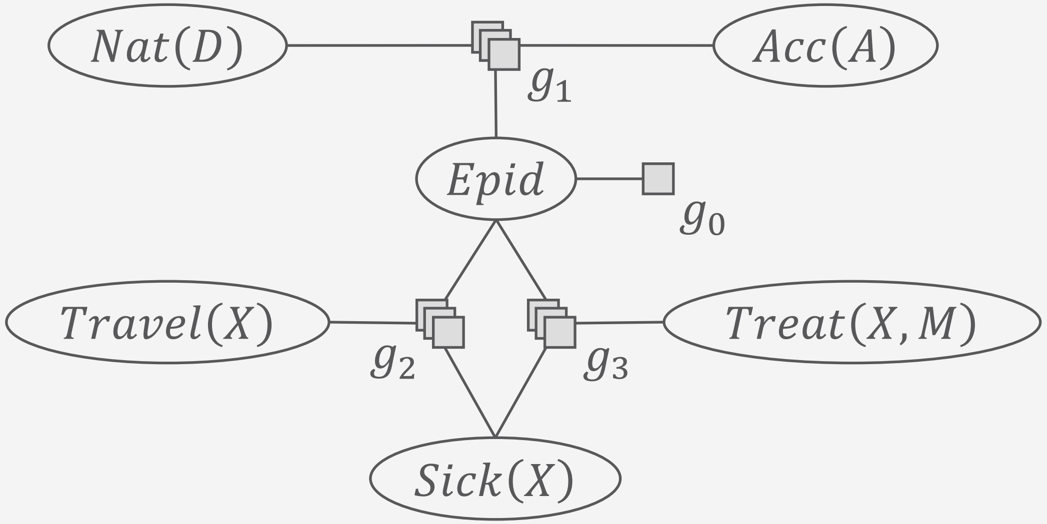

Factors

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$true$	$false$	$true$	6
$true$	$true$	$false$	2
$true$	$true$	$true$	9

$Travel(eve)$	$Epid$	$Sick(eve)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$fals$		
$true$	tru		
$true$	tru		

$Travel(bob)$	$Epid$	$Sick(bob)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
		$true$	6
		$false$	4
		$true$	6
		$false$	2
		$true$	9

$Travel(alice)$	$Epid$	$Sick(alice)$	g_2
$false$	$false$	$false$	5
$false$	$false$	$true$	0
$false$	$true$	$false$	4
$false$	$true$	$true$	6
$true$	$false$	$false$	4
$true$	$false$	$true$	6
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$reat(X, M)$

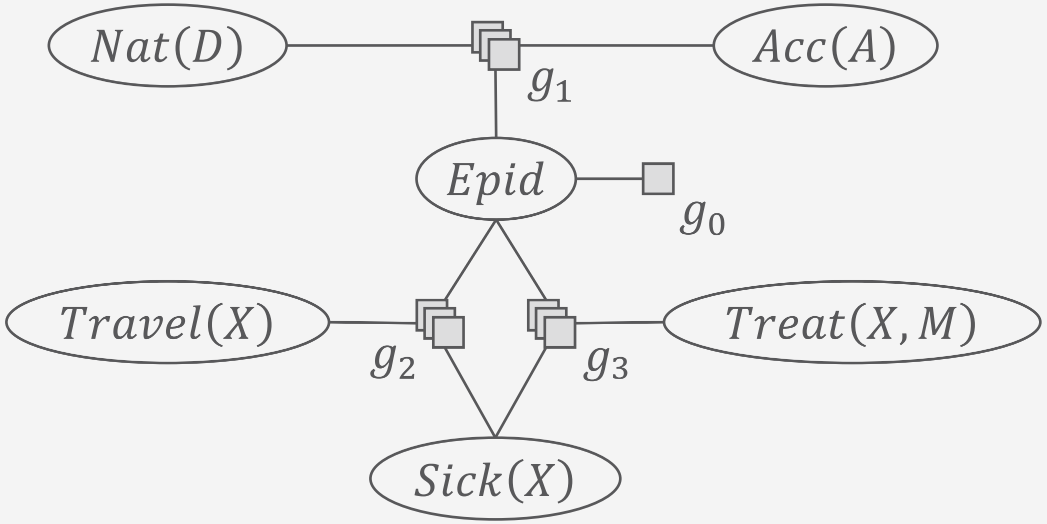
Encoding the Joint Distribution

- Set of parfactors = model
  - E.g., $G = \{g_1, g_2, g_3\}$
  - Semantics: Joint probability distribution P_G
    - Build by grounding, multiplying all grounded factors, and normalising the result
    - Grounding semantics [Sato 95, Fuhr 95]

$$P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$$

$$Z = \sum_{v \in r(rv(gr(G)))} \prod_{f \in gr(G)} f_i(\pi_{rv(f_i)}(v))$$

$\pi_{variables}(v)$ = projection of v onto $variables$



Encoding the Joint Distribution

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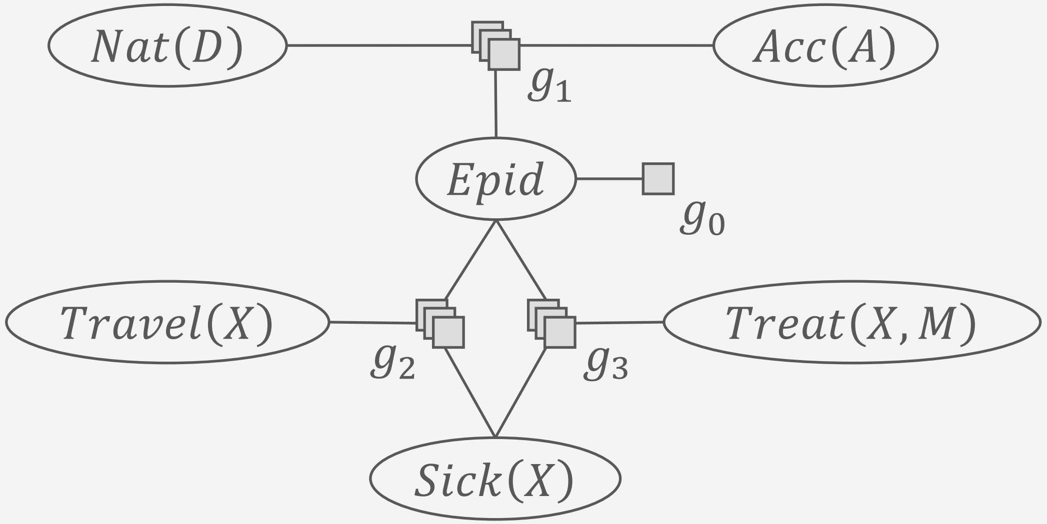
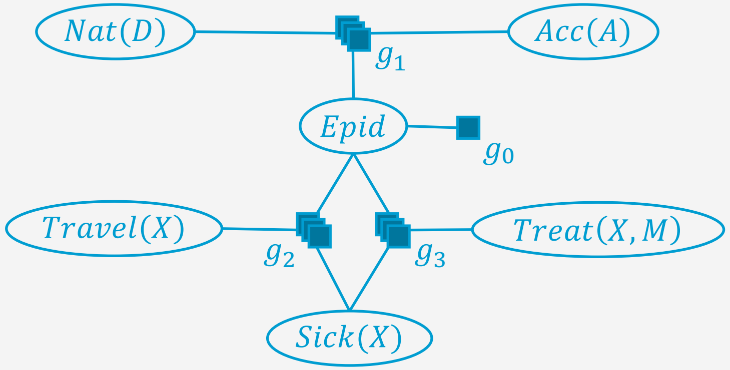
Sparse encoding of joint distribution

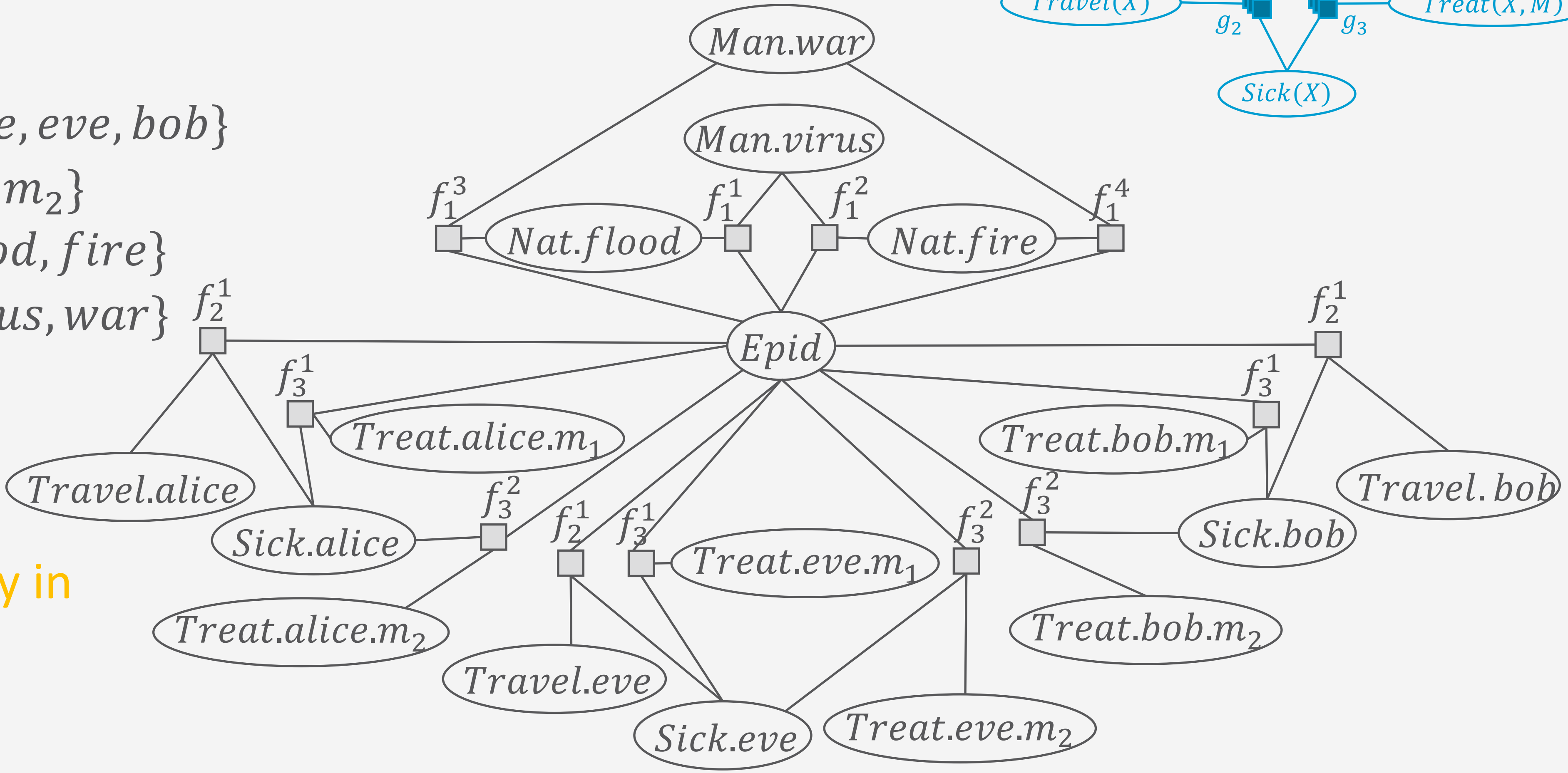
$3 \cdot 2^3 = 24$ entries in 3 parfactors, 6 PRVs



Grounded Model

- Given domains
  - $dom(X) = \{alice, eve, bob\}$
  - $dom(M) = \{m_1, m_2\}$
  - $dom(D) = \{flood, fire\}$
  - $dom(W) = \{virus, war\}$





- Indistinguishability in
  - Graph structure
  - Factors

Probabilistic Relational Models and Variants

- Parfactors Models
  [Poole 03, Taghipour et al. 13, B & Möller 16-19, Gehrke, B & Möller 18-19]

- Markov Logic Networks (MLNs) [Richardson & Domingos 06]
  - Use logical formulas to specify potential functions

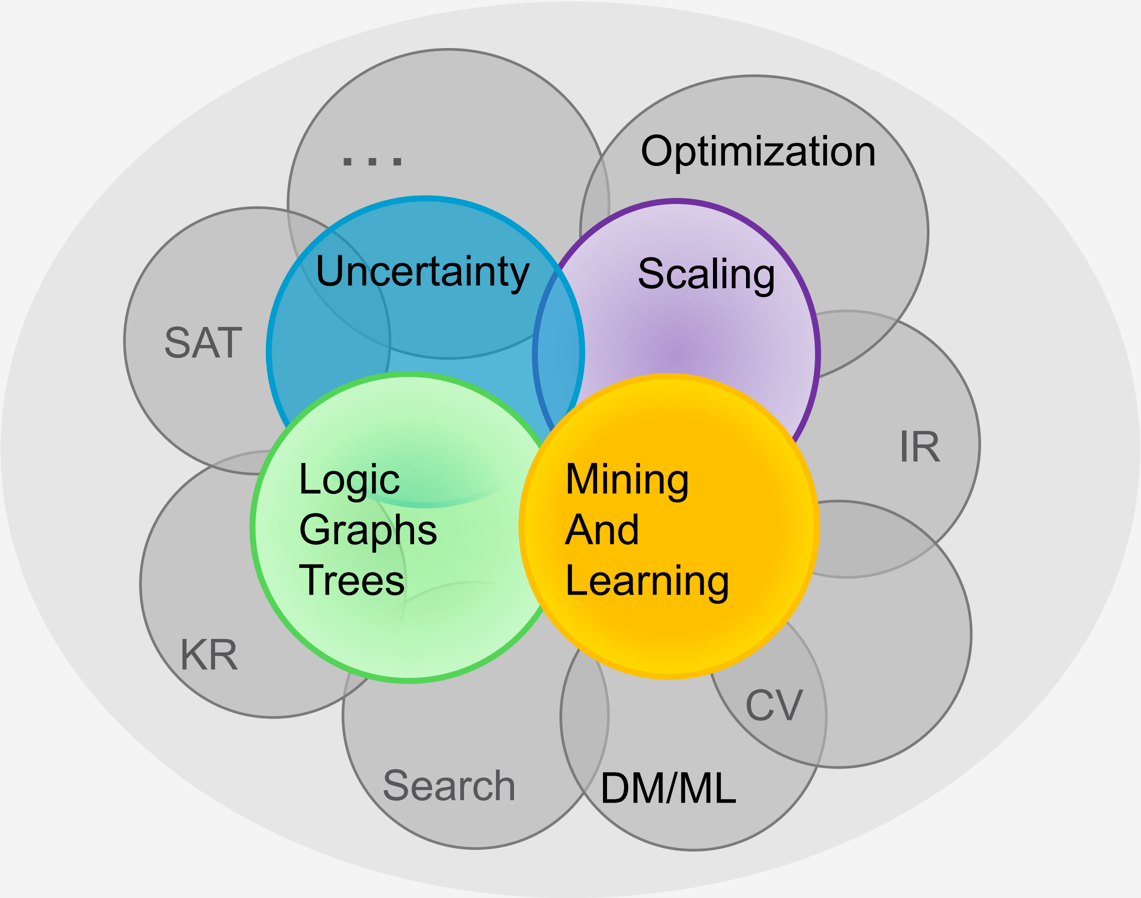
- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Use density functions to specify potential functions

- Based on grounding semantics [Sato 95, Fuhr 95]

The Larger Scope

Statistical Relational Learning & AI

- Study and design
  - intelligent agents
  - that reason about and
  - act in noisy worlds
  - composed of objects and relations among the objects



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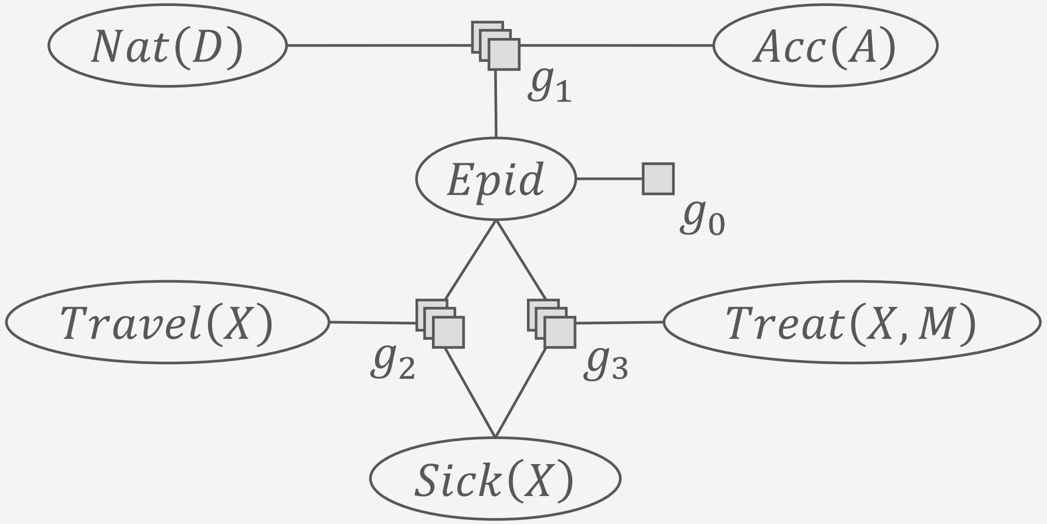
Lifted Query Answering and Tractability

The Power of Indistinguishability

Reasoning on Probabilistic Relational Models

- Inference task: query answering (QA)
- Queries:
  - Marginal distribution
    - $P(Sick(eve))$
    - $P(Travel(eve,)\ Treat(eve, m_1))$
  - Conditional distribution
    - $P(Sick(eve) | Epid)$
    - $P(Epid | Sick(eve) = true)$
  - Assignment queries: $\arg\max_{\boldsymbol{a} \in ran(\boldsymbol{A})} P(\boldsymbol{a} | \boldsymbol{e})$
    - MPE: $\boldsymbol{A} = rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
    - MAP: $\boldsymbol{A} \subseteq rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
      - What is not in $\boldsymbol{A}$ needs to be summed out



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- Queries:
  - Marginal distribution
    - $P(Sick(eve))$
    - $P(Travel(eve,)\ Treat(eve, m_1))$
  - Conditional distribution
    - $P(Sick(eve)|Epid)$
    - $P(Epid|Sick(eve) = true)$
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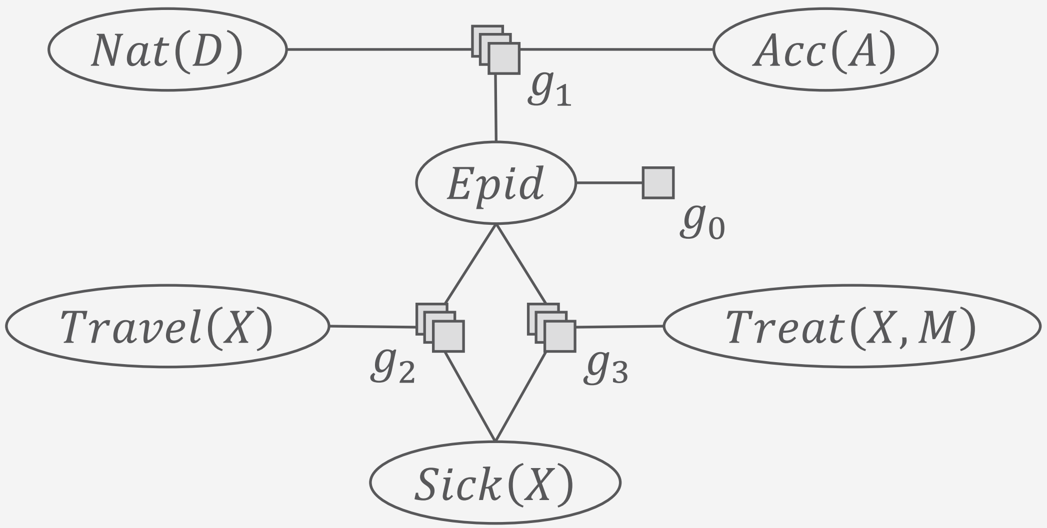
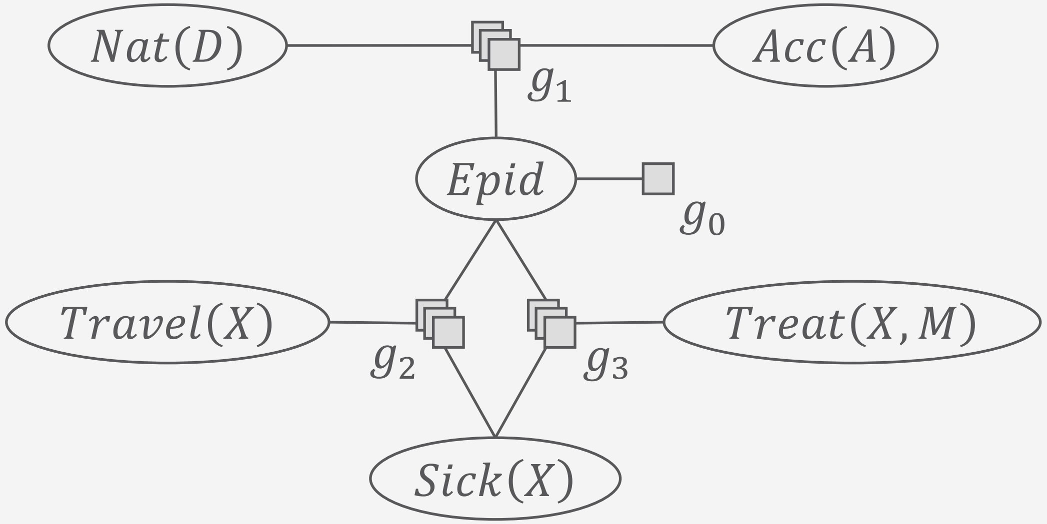
    - MAP: $\boldsymbol{A} \subseteq rv(\boldsymbol{G}) \setminus rv(\boldsymbol{e})$
      - What is not in $\boldsymbol{A}$ needs to be summed out

Goal: Avoid groundings!
→ *lifted* inference



QA: Lifted Variable Elimination (LVE)

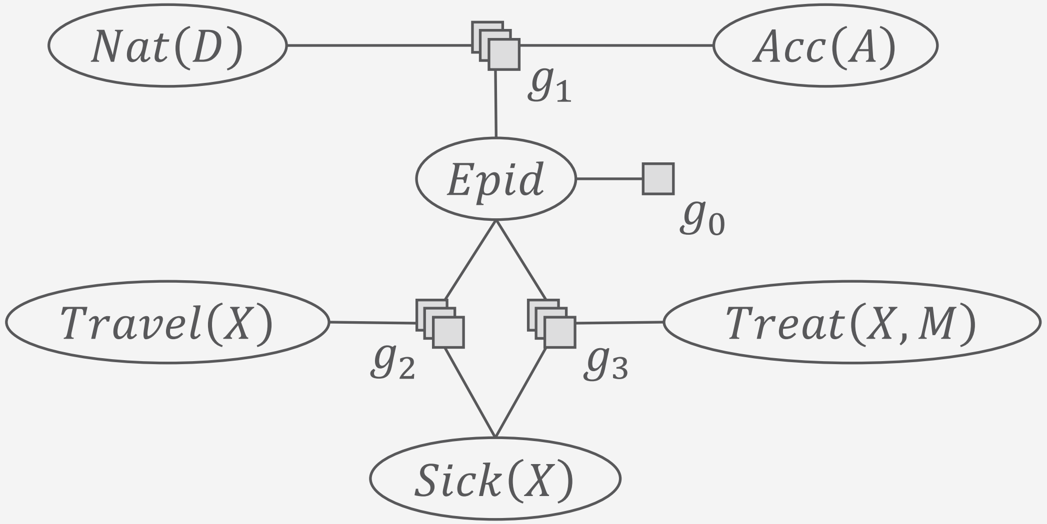
- Eliminate all variables not appearing in query
- Lifted summing out
  - Sum out *representative* instance as in propositional variable elimination
  - Exponentiate result for indistinguishable instances



[Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a]

QA: Lifted Variable Elimination (LVE)

- Eliminate all variables not appearing in query
- Lifted summing out
  - Sum out *representative* instance as in propositional variable elimination
  - Exponentiate result for indistinguishable instances
- Correctness: Equivalent ground operation
  - Each instance is summed out
  - Result: factor f that is identical for all instance
  - Multiplying indistinguishable results
    → exponentiation of one representative f



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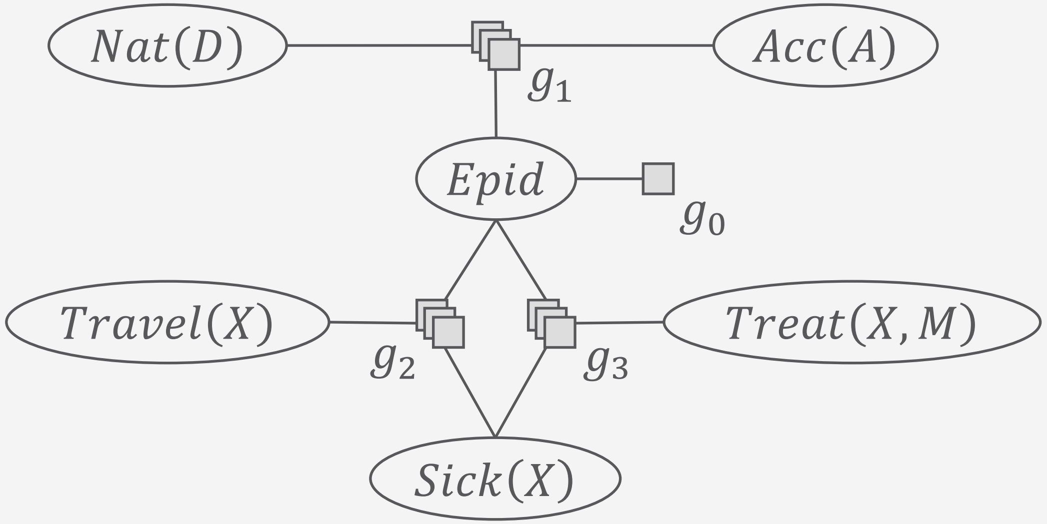
QA: LVE in Detail

- E.g., marginal
  - $P\big(Travel(eve)\big)$
  - Split atoms $R(\dots, X, \dots)$ w.r.t. eve if eve in $dom(X)$



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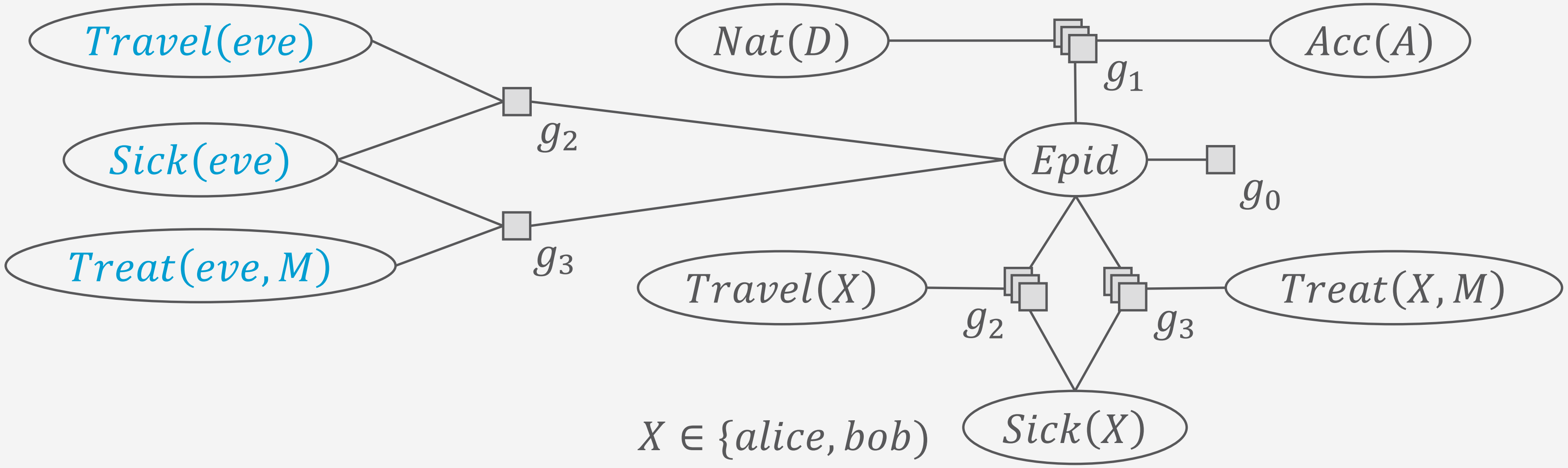

QA: LVE in Detail

- E.g., marginal
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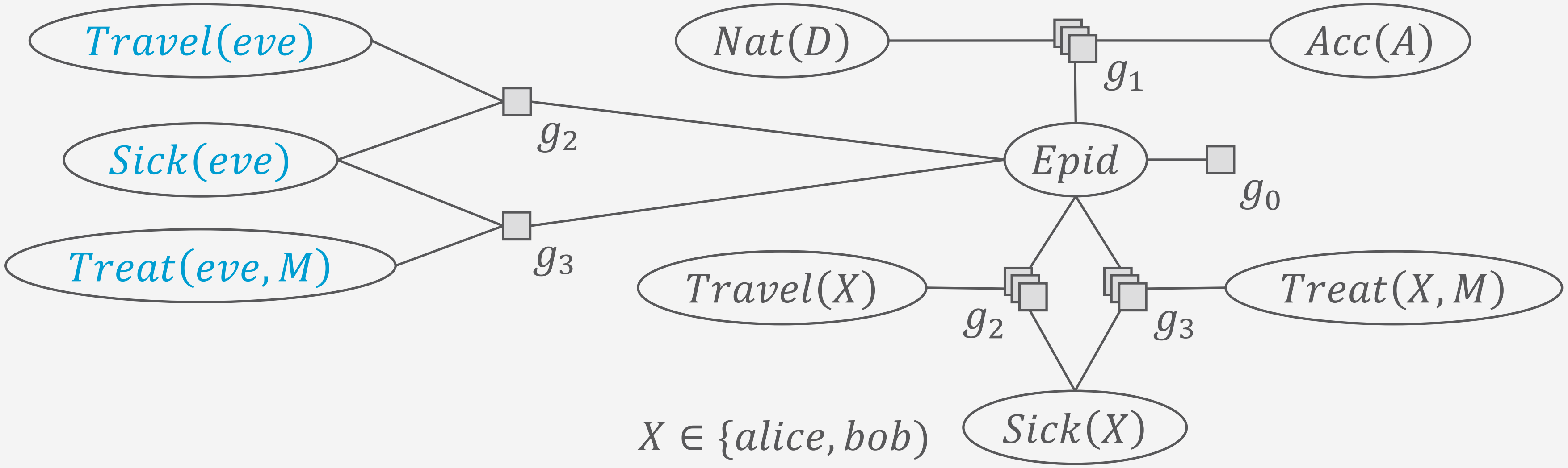
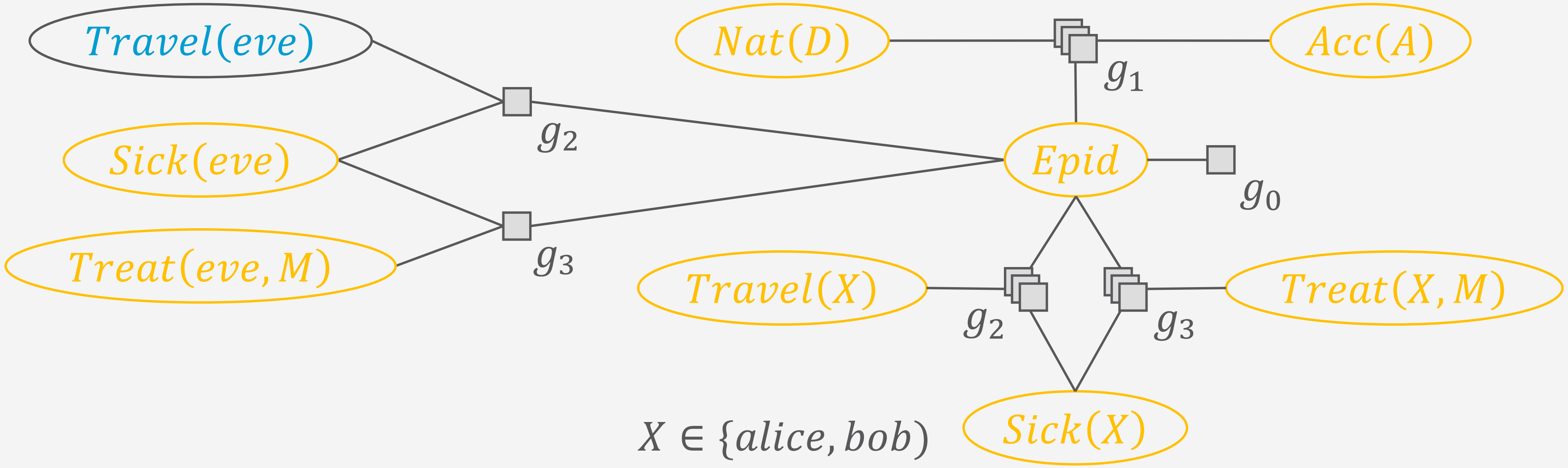
Shattering



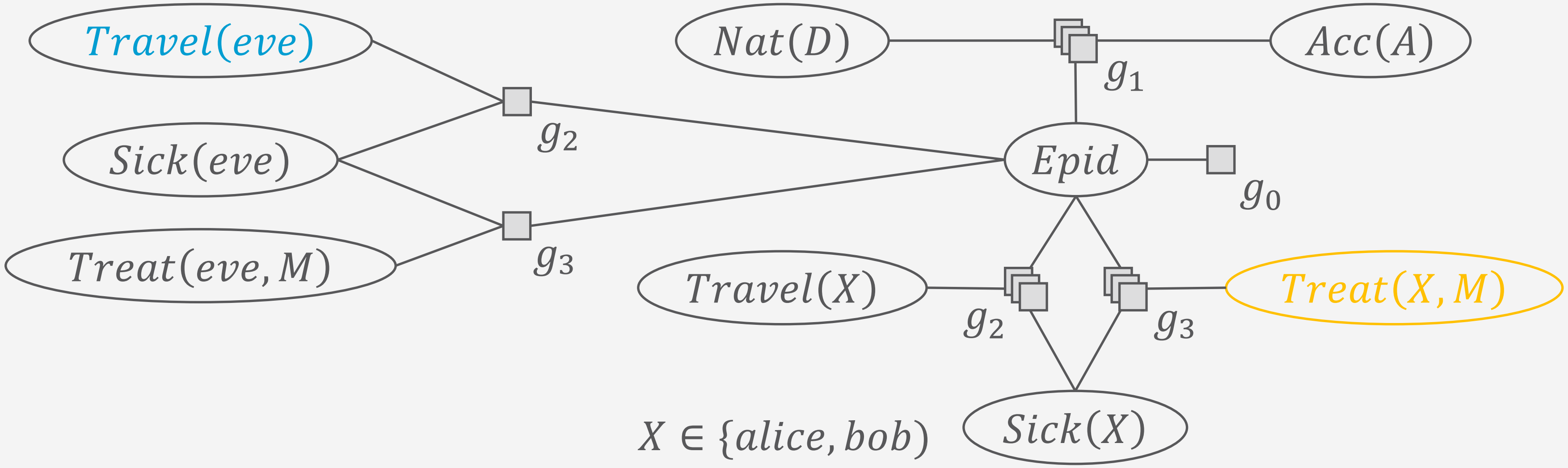
QA: LVE in Detail

- E.g., marginal
  - $P(Travel(eve))$
  - Split atoms $R(\dots, X, \dots)$ w.r.t. eve if eve in $dom(X)$
  - Eliminate all non-query variables



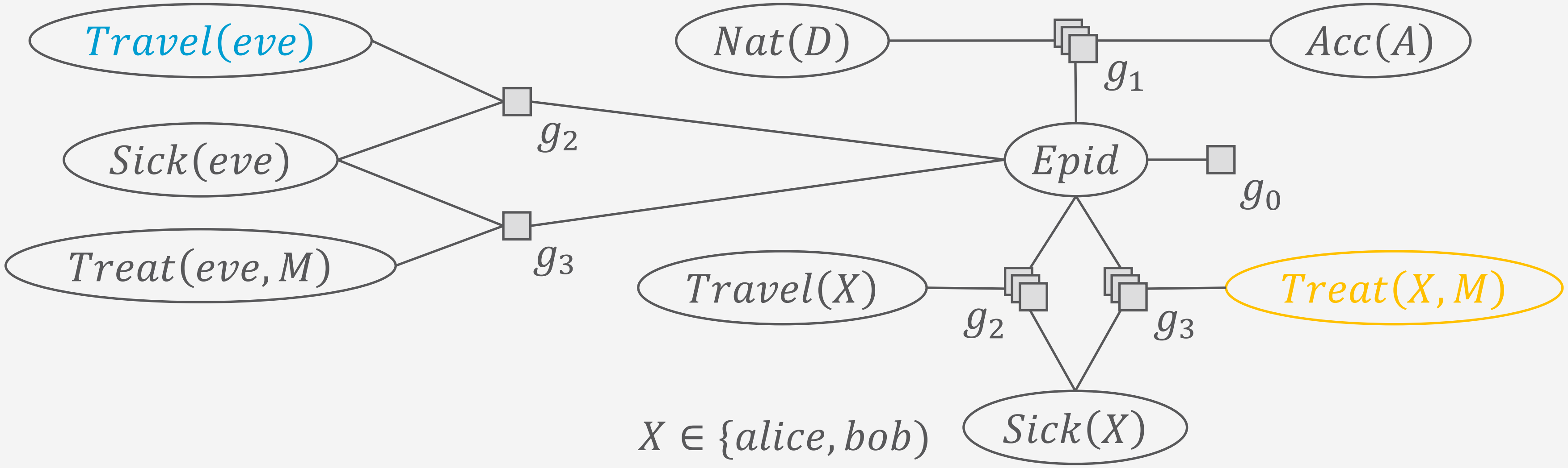
QA: LVE in Detail

- Eliminate $Treat(X, M)$



QA: LVE in Detail

- Eliminate $Treat(X, M)$
  - Appears in only one g: g_3
  - Contains all logical variables of g_3: X, M
  - For each X constant: the same number of M constants



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- Eliminate $Treat(X, M)$
  - Appears in only one g: g_3
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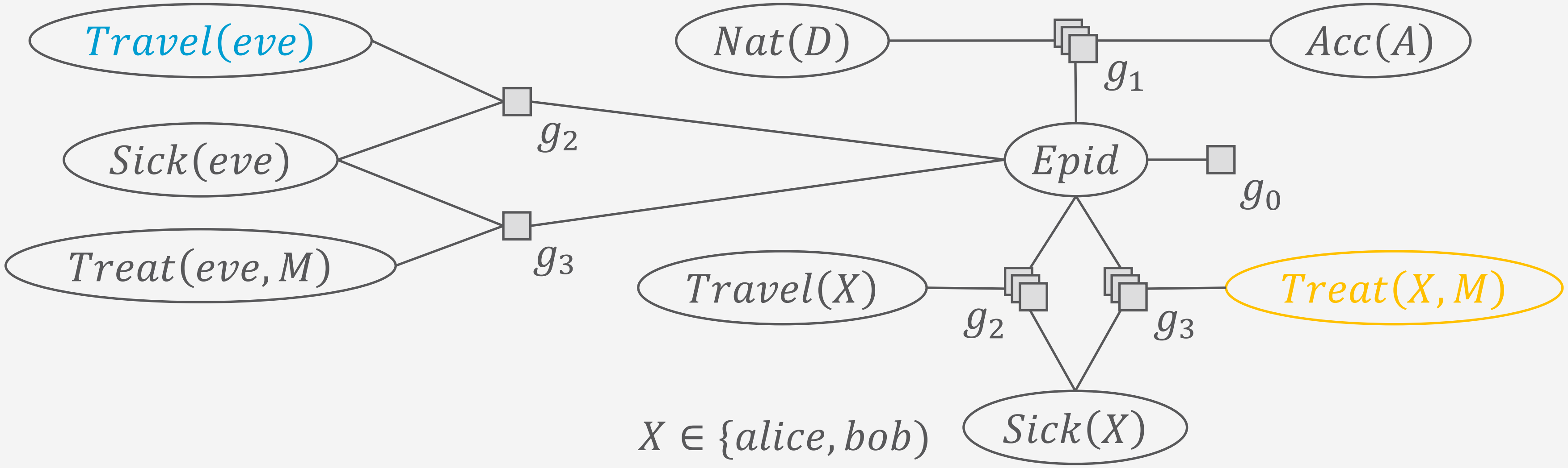
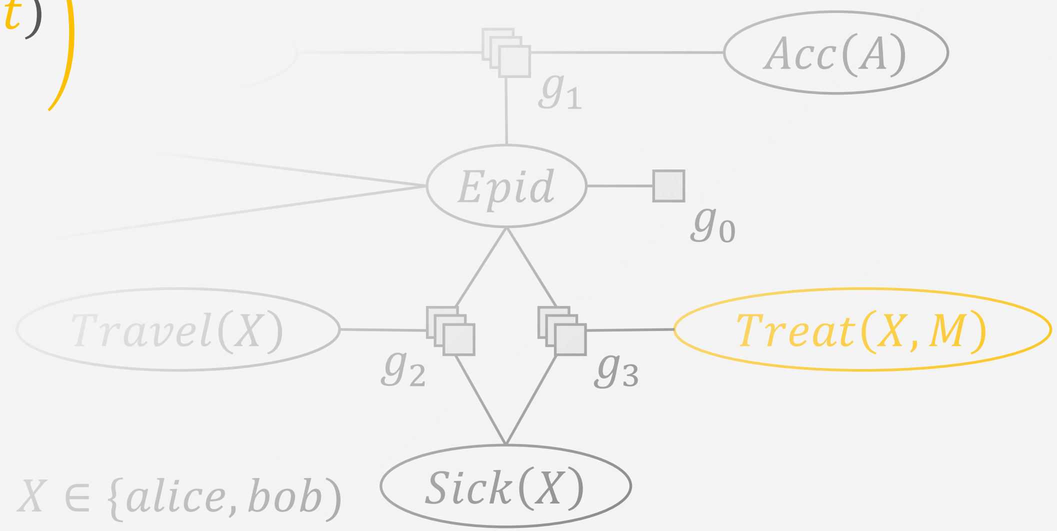
  - For each X constant: the same number of M constants
  - ✓ Preconditions of lifted summing out fulfilled, lifted summing out possible



LVE in Detail: Lifted Summing Out

- Eliminate $Treat(X, M)$ by lifted summing out
  1. Sum out representative
  2. Exponentiate for indistinguishable objects

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t)\right)^{\#M|X}$$



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$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t)\right)^{\#M|X}$$

$Epid$	$Sick(X)$	$Treat(X,M)$	g_3
$false$	$false$	$false$	9
$false$	$false$	$true$	1
$false$	$true$	$false$	6
$false$	$true$	$true$	3
$true$	$false$	$false$	7
$true$	$false$	$true$	5
$true$	$true$	$false$	4
$true$	$true$	$true$	8

+

$Epid$	$Sick(X)$	Σ
$false$	$false$	10

LVE in Detail: Lifted Summing Out

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X,M) = t)\right)^{\#M|X}$$

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$true$	$true$	$true$	8

$Epid$	$Sick(X)$	Σ
$false$	$false$	10
$false$	$true$	9
$true$	$false$	12
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+ + + +

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$true$	$true$	12

^ ^ ^ ^

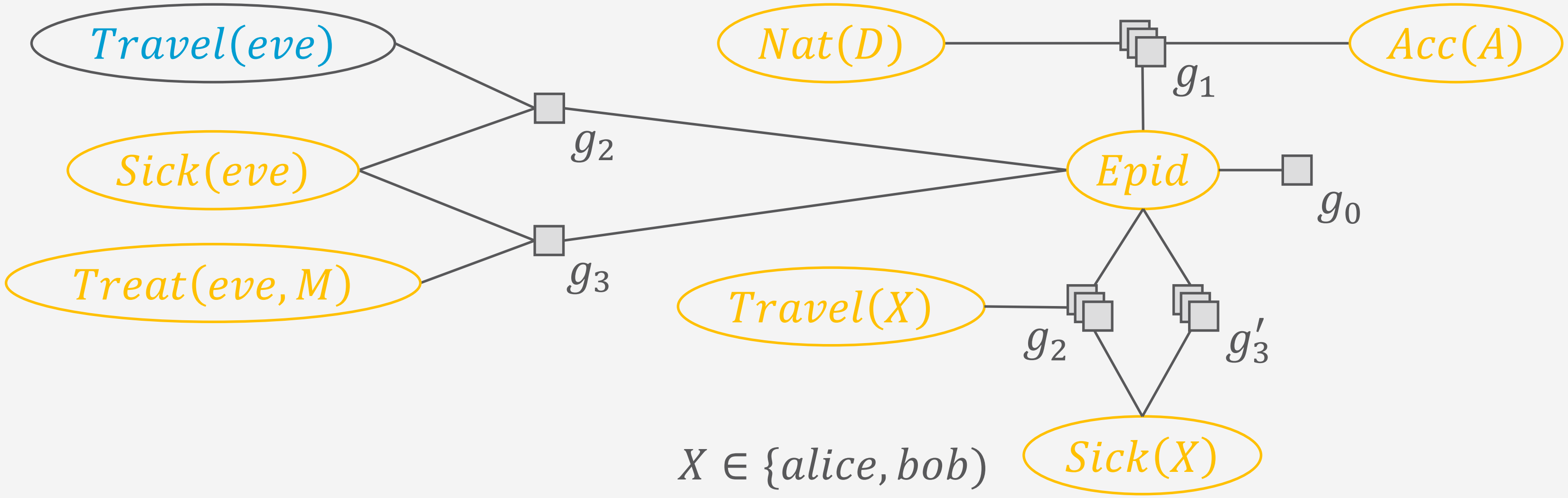
$Epid$	$Sick(X)$	^
$false$	$false$	10^2
$false$	$true$	9^2
$true$	$false$	12^2
$true$	$true$	12^2

LVE in Detail: Lifted Summing Out

- Result after summing out $Treat(X, M)$

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t) \right)^{\#M|X}$$

$Epid$	$Sick(X)$	g_3'
$false$	$false$	100
$false$	$true$	81
$true$	$false$	144
$true$	$true$	144



LVE in Detail: Lifted Summing Out

Only here, domain size comes into play → no change in graph / parfactor if domain size changes

- Result after summing out $Treat(X, M)$

$$\left(\sum_{t \in r(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t) \right)^{\#M|X}$$

$Epid$	$Sick(X)$	g'_3
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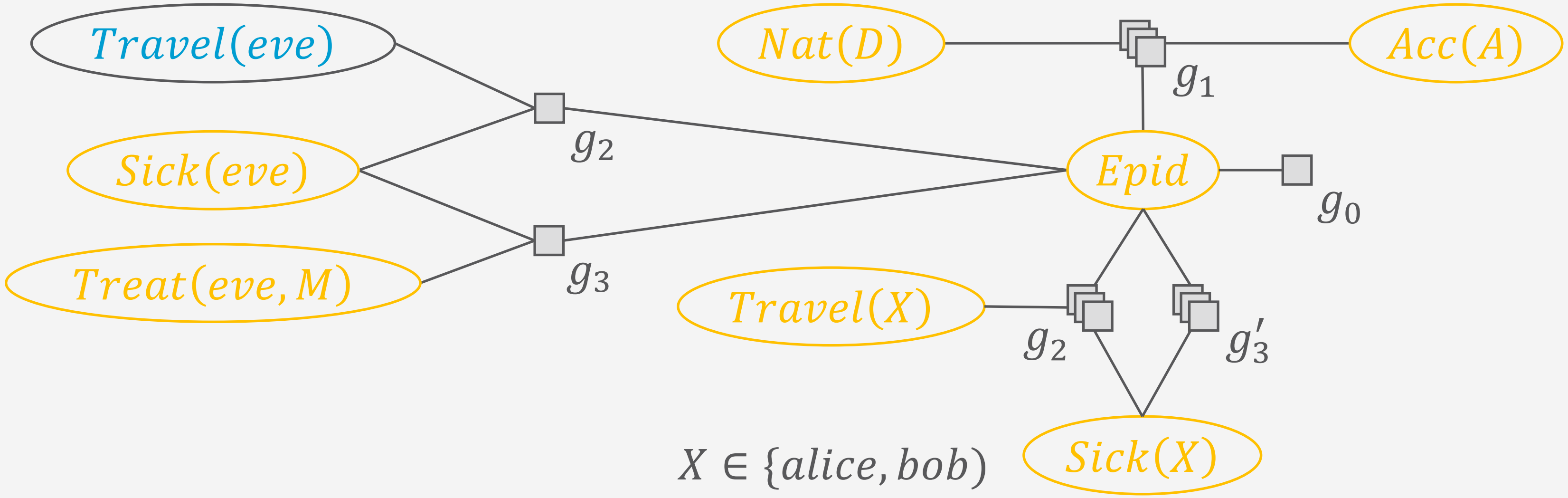

Tractability

- Given a model that allows for lifted calculations
  - I.e., no groundings during solving an instance of the problem
- Solving an instance of the problem is possible in time polynomial in domain sizes

  → The query answering algorithm is domain-lifted
- An query answering problem is tractable
  - when it is solved by an efficient algorithm, running in time polynomial in the number of random variables
- Assume that the number of random variables is characterised by domain sizes
  - Then, solving a query answering problem is tractable under domain-liftability
    - Runtime might still be exponential in other terms
    - More general results by Niepert & Van den Broeck (2014)

Indistinguishable Evidence and Query Terms

Evidence

- Observations for instances of a PRV
  - One of the range values
  - Not available
- Treat as groups per observation
  - Shatter model on the groups
- Example: 10 instances observed true

$Sick(X^T)$	g_e^T
$false$	0
$true$	1

$dom(X^T) = \{x_1, \dots, x_{10}\}$
$dom(X) = \{x_{11}, \dots, x_n\}$

Indistinguishable Evidence and Query Terms

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$false$	0
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$dom(X) = \{x_{11}, \dots, x_n\}$

Query Terms

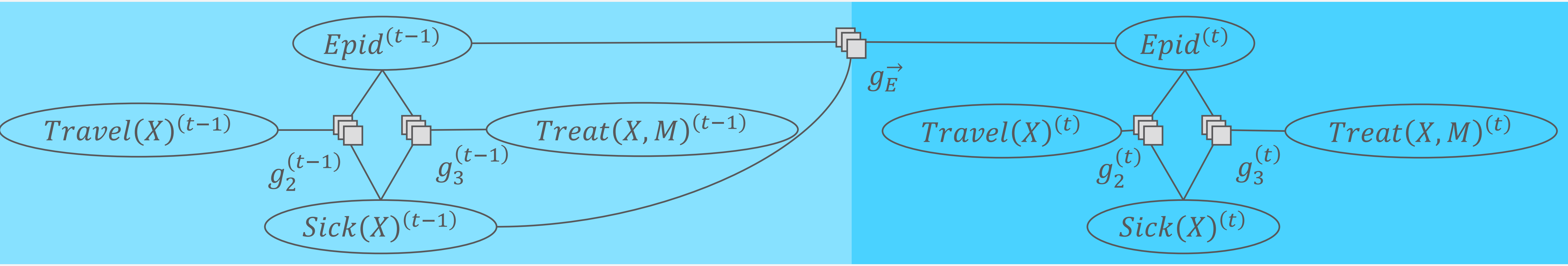
- Indistinguishable instances in query:
  - $P\big(Sick(alice), Sick(eve), Sick(bob)\big)$
  - Result will have local symmetries, e.g., 2 false and 1 true maps to potential of 2
- Parameterised query: $P\big(Sick(X)\big)$
- Use standard LVE
  - Count conversion yields wanted result

$\#_X[Sick(X)]$	g
[0,3]	1
[1,2]	2
[2,1]	3
[3,0]	4

Keeping Indistinguishability over Time

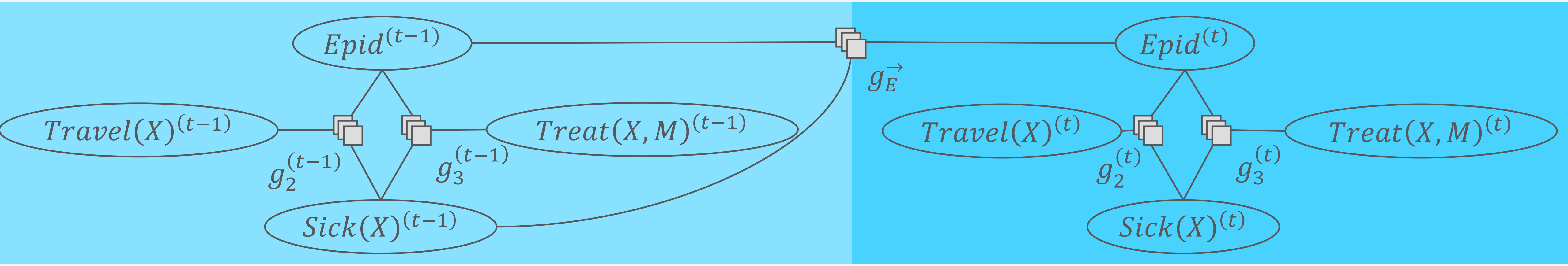
The Power of Indistinguishability

Dynamic Probabilistic Relational Models & Temporal Queries



- Marginal distribution queries: $P\left(A_\pi^i \mid E_{0:t}\right)$
  - Hindsight: $\pi < t$ (Was there an epidemic $t - \pi$ days ago?)
  - Filtering: $\pi = t$ (Is there currently an epidemic?)
  - Prediction: $\pi > t$ (Will there be an epidemic in $\pi - t$ days?)
- Assignment queries on temporal sequence

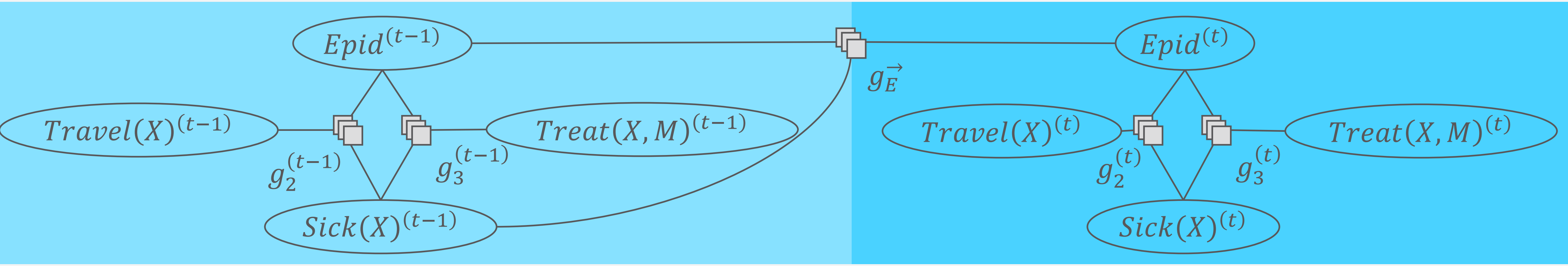
Reasoning over Time: Interfaces



- Main idea: Use temporal conditional independences for efficient temporal QA
  - Normally only a subset of random variables influence next time step → interface variables
  - State description of interface from time slice $t-1$ suffices to perform inference on time slice t
  - → Makes present independent from past / future

Algorithms:
- Propositional: Interface Algorithm [Murphy, 2002]
- Lifted: Lifted Dynamic Junction Tree Algorithm [Gehrke et al, 2018]

Taming Reasoning

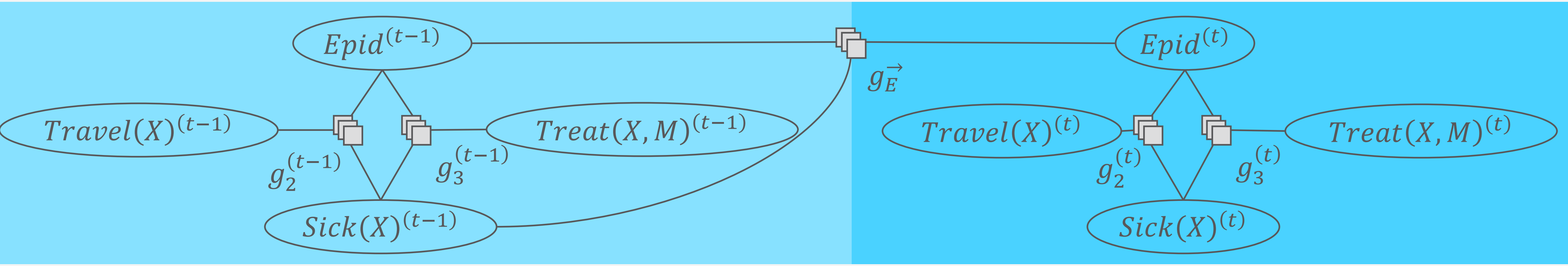


- Evidence can ground a model over time
- Non-symmetric evidence
  - Observe evidence for some instances in one time step
  - Observe evidence for a subset of these instances in another time step
  - Split the logical variable slowly over time

Interface carries over splits, leading to slowly grounding a model over time

Gehrke et al. (2020)

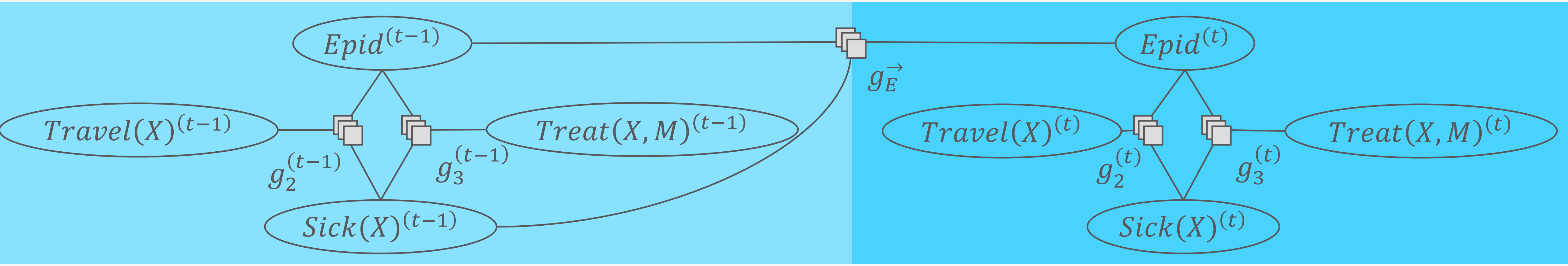
Undoing Splits



- Need to undo splits to
  keep reasoning polynomial w.r.t. domain sizes
- Where can splits be undone efficiently?
  - When moving from one time step to the next, i.e., in the interface
- How to undo splits?
  - Find approximate symmetries
  - Merge based on groundings
- Is it reasonable to undo splits?
  - Effect of slight differences in evidence?
  - Impact of evidence vs. temporal model

Gehrke et al. (2020)

Is It Reasonable to Undo Splits?



- Approximate forward message
- For each time step the temporal behaviour is multiplied on the forward message
- Indefinitely bounded error due to temporal behaviour

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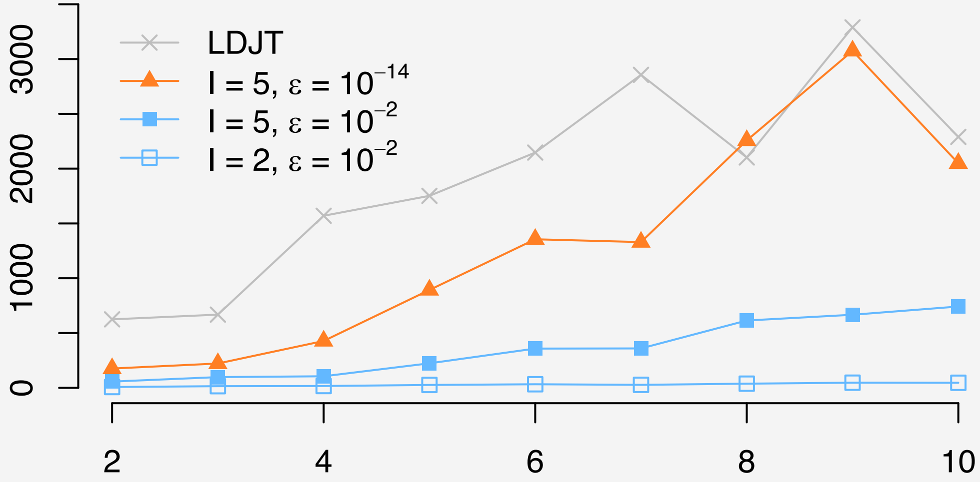
Results

- DBSCAN for Clustering
- ANOVA for checking fitness of clusters
- Right: runtimes
- Below: approximation error



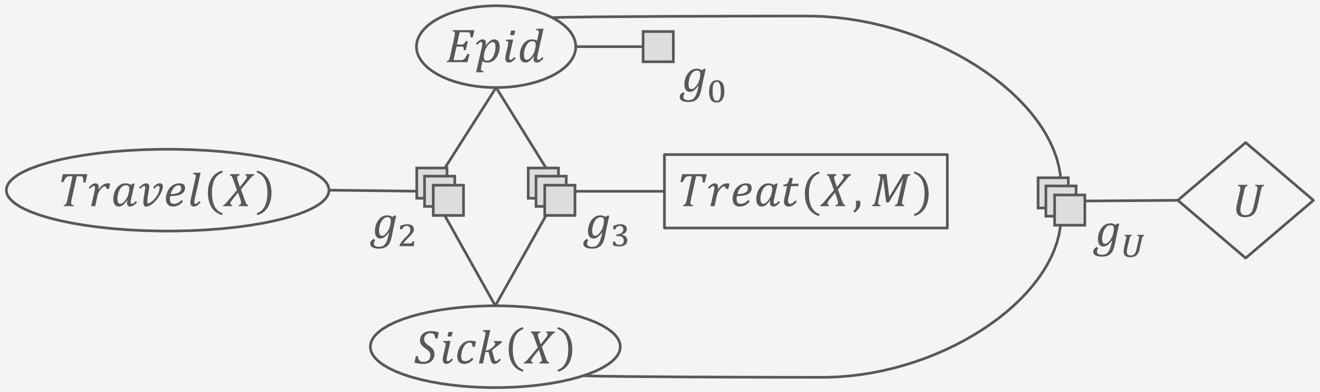
π	Max	Min	Average
0	0.0001537746121	0.0000000001720	0.0000191206488
2	0.0000000851654	0.0000000000001	0.0000000111949
4	0.0000000000478	0	0.0000000000068

Indistinguishability in Decision Making

The Power of Indistinguishability

Indistinguishability for Decision Making

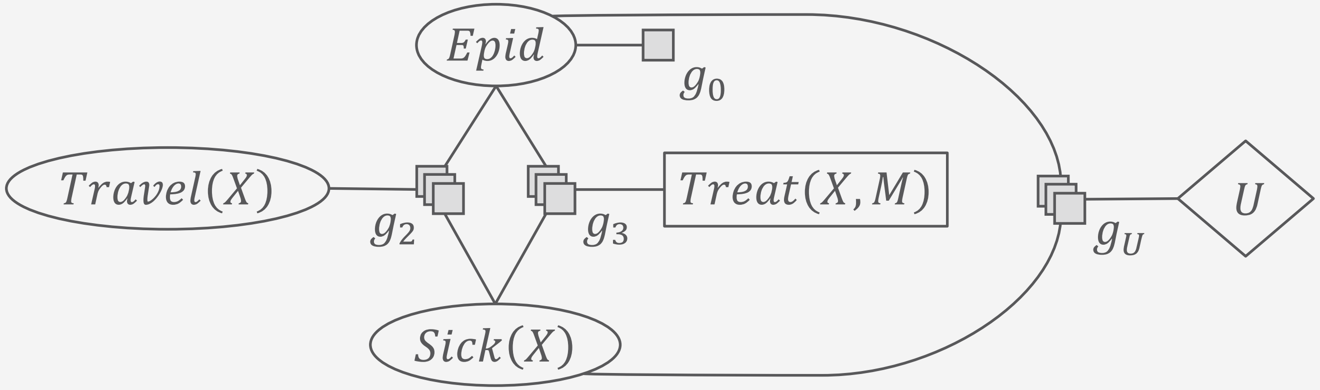
- Online decision making: Graphical models extended by decision and utility nodes
  - Parameterise decisions to make decisions for whole groups of indistinguishable instances: $Treat(X, M)$ (box in graph)
  - PRVs in utility functions to denote identical share in contributed utility U (diamond in graph) : $\phi_U\big(Epid, Sick(X)\big)$
  - (Dynamic) decision parfactor models, Markov logic decision networks



Indistinguishability for Decision Making

- Inference task: maximum expected utility (MEU) query
  - *Which actions can be expected to lead to the maximum utility?*
  - Standard inference algorithms more or less still work
    - Iterate through all possible decisions, set decisions as evidence, calculate expected utility, store current maximum
    - Solve an MAP query with decision variables as query terms and the other variables in the model to eliminate

Assign same action to group of indistinguishable instances
- Fewer possible decisions to consider → *tractability*!



Indistinguishability for Decision Making

- Offline decision making: solve a (partially observable) Markov decision problem (POMDP)
  - First-order / relational MDPs: indistinguishability in the environment [Sanner & Kersting 2012]
    - Based on situation calculus: work with representatives
      - E.g., it is important that <u>a</u> box with medical supplies arrives at a destination but not which one it is in particular (of a set of boxes with medical supplies)
    - Novel propositional situations worth exploring may be instances of a well-known context in the relational setting → *exploitation* promising
      - E.g., household robot learning water-taps
      - Having opened one or two water-taps in a kitchen, one can expect other water-taps in kitchens to work similarly

      ⇒ Priority for exploring water-taps in kitchens in general reduced

      ⇒ Information gathered likely to carry over to water-taps in other places

      ❖ Hard to model in propositional setting: each water-tap is novel

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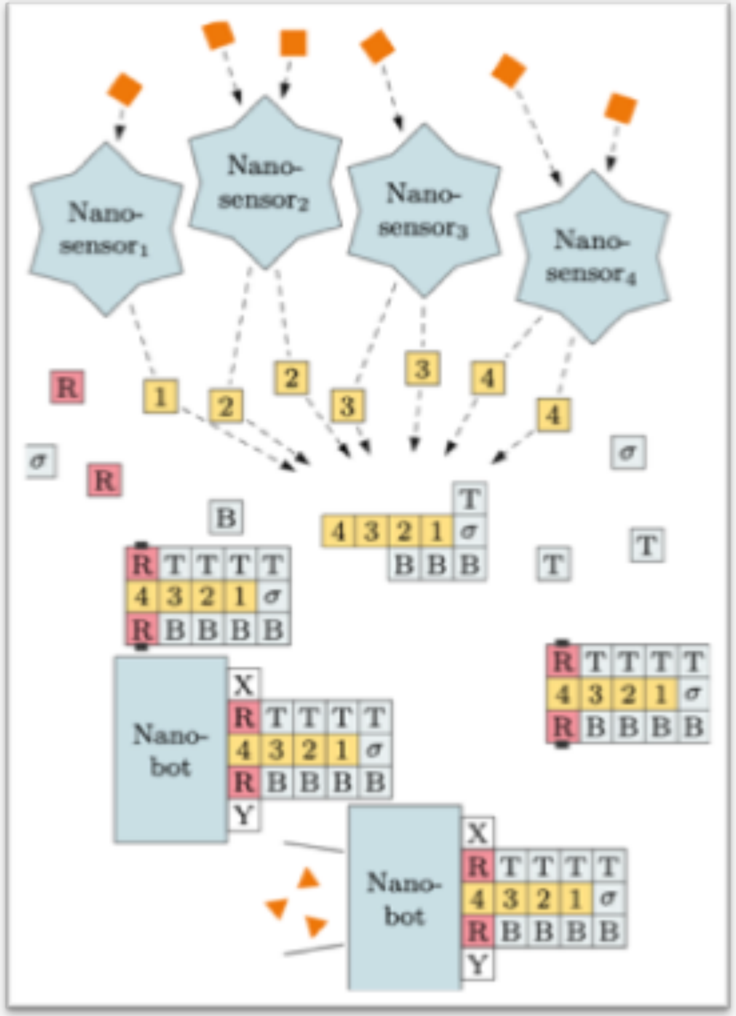
Indistinguishability for Decision Making

- Multi-agent setting: decentralised POMDP [Oliehoek & Amato 2016]
  - Set of agents with
    - Own set of available actions, observations
    - *Shared* state and reward
- Lifting for agents [B et al. 2022]
  - Agents with indistinguishable behaviour → types
    - The same sets of actions, observations available
    - Same strategy / program applies if certain independences hold
  - Groups by types can be treated by representatives
    - Reduce exponential dependence on agent numbers
  - Application: Nanoagent network



Agenda

- Statistical Relational Artificial Intelligence
  - Probabilistic relational models
  - Grounding semantics
  - Context
- The Power of Indistinguishability
  - Lifted query answering and tractability
  - Keeping indistinguishability over time
  - Indistinguishability in decision making
- Summary

The Finish Line: The Power of Indistinguishability

- Lifted query answering and tractability
  - Use information about indistinguishability to speed up inference
  - Tractability in terms of domain sizes through lifting
  - Handle evidence in groups of indistinguishable observations
  - Count values in histograms for lifted queries
- Keeping indistinguishability over time
  - Merge parfactors with bounded error
- Indistinguishability in decision making
  - Relational environment encoded
  - Agent types

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What else is there to do? – Oh, so much…
- Approximating symmetries
- Generalising lifting operators
- More robust learning algorithms
- Privacy
- Ethical behaviour
- Explainability
- …

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Thank you!

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Ordered topic-wise and then alphabetically

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