A Glimpse into Statistical Relational AI
The Power of Indistinguishability
Tanya Braun, University of Münster
Agenda

• Statistical Relational Artificial Intelligence
  • Probabilistic relational models
  • Grounding semantics
  • Context
• The Power of Indistinguishability
  • Lifted query answering and tractability
  • Keeping indistinguishability over time
  • Indistinguishability in decision making
• Summary
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• Summary
Statistical Relational Artificial Intelligence (StaRAI)

AI: intelligent systems in the real world

Figure based on Stuart Russell
**Statistical Relational Artificial Intelligence (StaRAI)**

The world has things in it!

The world is uncertain!

First-order logic

AI: intelligent systems in the real world

Probabilistic graphical models

The world has things in it!

The world is uncertain!

Statistical Relational Artificial Intelligence

Figure based on Stuart Russell
Statistical Relational Artificial Intelligence (StaRAI)

The world has things in it!
The world is uncertain!
First-order logic

AI: intelligent systems in the real world
The world is uncertain!
Probabilistic graphical models

The world has things in it!
Statistical Relational Artificial Intelligence

Probabilistic relational models

Figure based on Stuart Russell
Application: Epidemics

• Atoms: Parameterised random variables = PRVs
  • With logical variables
    • E.g., $X$, $M$
    • Possible values (domain):
      \[
      \text{dom}(X) = \{\text{alice, eve, bob}\}
      \text{dom}(M) = \{\text{injection, tablet}\}
      \]
  • With range
    • E.g., Boolean
    • $\text{ran(Travel}(X)) = \{\text{true, false}\}$
  • Represent sets of indistinguishable random variables

Nat($D$) = natural disaster $D$
Acc($A$) = accident $A$

\[
\text{Nat}(D) \quad \text{Acc}(A) \quad \text{Epid} \quad \text{Travel}(X) \quad \text{Treat}(X, M) \quad \text{Sick}(X)
\]
Encoding the Joint Distribution: Factorisation

• Factors with PRVs = **parfactors**
  • E.g., $g_2$

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<th>Epid</th>
<th>Sick($X$)</th>
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**Potentials**
• In parfactors, just like in factors, no probability distribution as factors required
Factors

• Grounding
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

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Factors

- **Grounding**
  - E.g., $gr(g_2) = \{f_2^1, f_2^2, f_2^3\}$

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Treat($X, M$)
Encoding the Joint Distribution

- Set of parfactors = model
  - E.g., $G = \{g_1, g_2, g_3\}$
- Semantics: Joint probability distribution $P_G$
  - Build by grounding, multiplying all grounded factors, and normalising the result
  - Grounding semantics [Sato 95, Fuhr 95]

$$P_G = \frac{1}{Z} \prod_{f \in gr(G)} f$$

$$Z = \sum_{\nu \in rv(gr(G))} \prod_{f \in gr(G)} f_i(\pi_{rv(f_i)}(\nu))$$

$\pi_{variables}(\nu)$ = projection of $\nu$ onto variables
Encoding the Joint Distribution

• Set of parfactors = \textit{model}
  • E.g., \( G = \{g_1, g_2, g_3\} \)
  • Semantics: Joint probability distribution \( P_G \)
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\[
P_G = \frac{1}{Z} \prod_{f \in \text{gr}(G)} f
\]
\[
Z = \sum_{\nu \in \text{rv(gr}(G)))} \prod_{f \in \text{gr}(G)} f_i(\pi_{\text{rv}(f_i)}(\nu))
\]
\( \pi_{\text{variables}}(\nu) = \text{projection of } \nu \text{ onto } \text{variables} \)

Sparse encoding of joint distribution

\( 3 \cdot 2^3 = 24 \) entries in 3 parfactors, 6 PRVs
Grounded Model

• Given domains
  • \( \text{dom}(X) = \{\text{alice, eve, bob}\} \)
  • \( \text{dom}(M) = \{m_1, m_2\} \)
  • \( \text{dom}(D) = \{\text{flood, fire}\} \)
  • \( \text{dom}(W) = \{\text{virus, war}\} \)

• Indistinguishability in
  • Graph structure
  • Factors

\[ \begin{align*}
\text{Nat}(D) &\xrightarrow{g_1} \text{Acc}(A) \\
\text{Epid} &\xrightarrow{g_0} \text{Treat}(X, M) \\
\text{Travel}(X) &\xrightarrow{g_2} \text{Sick}(X) \\
\end{align*} \]
Probabilistic Relational Models and Variants

- Parfactors Models
  [Poole 03, Taghipour et al. 13, B & Möller 16-19, Gehrke, B & Möller 18-19]

- Markov Logic Networks (MLNs) [Richardson & Domingos 06]
  - Use logical formulas to specify potential functions

- Probabilistic Soft Logic (PSL) [Bach et al. 17]
  - Use density functions to specify potential functions

- Based on grounding semantics [Sato 95, Fuhr 95]
The Larger Scope

Statistical Relational Learning & AI

• Study and design
  • intelligent agents
  • that reason about and
  • act in noisy worlds
  • composed of objects and relations among the objects
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Lifted Query Answering and Tractability
The Power of Indistinguishability
Reasoning on Probabilistic Relational Models

- Inference task: query answering (QA)
- Queries:
  - **Marginal** distribution
    - $P(\text{Sick}($eve$))$
    - $P(\text{Travel}($eve, $)$ \text{Treat}($eve,m_1$))
  - **Conditional** distribution
    - $P(\text{Sick}($eve$)|\text{Epid})$
    - $P(\text{Epid}|\text{Sick}($eve$) = \text{true})$
  - **Assignment** queries: $\arg \max_{a \in \text{ran}(A)} P(a|e)$
    - **MPE**: $A = \text{rv}(G) \setminus \text{rv}(e)$
    - **MAP**: $A \subseteq \text{rv}(G) \setminus \text{rv}(e)$
      - What is not in $A$ needs to be summed out
Reasoning on Probabilistic Relational Models

• Inference task: query answering (QA)
• Queries:
  • Marginal distribution
    • $P(\text{Sick}(\text{eve}))$
    • $P(\text{Travel}(\text{eve},) \, \text{Treat}(\text{eve}, m_1))$
  • Conditional distribution
    • $P(\text{Sick}(\text{eve})|\text{Epid})$
    • $P(\text{Epid}|\text{Sick}(\text{eve}) = \text{true})$
  • Assignment queries: arg max $P(a|e)$
    • MPE: $A = \text{ran}(G) \setminus \text{rv}(e)$
    • MAP: $A \subseteq \text{ran}(G) \setminus \text{rv}(e)$
      • What is not in $A$ needs to be summed out

Goal: Avoid groundings! $\rightarrow$ lifted inference
QA: Lifted Variable Elimination (LVE)

• Eliminate all variables not appearing in query
• Lifted summing out
  • Sum out *representative* instance as in propositional variable elimination
  • Exponentiate result for indistinguishable instances

[Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a]
QA: Lifted Variable Elimination (LVE)

- Eliminate all variables not appearing in query
- Lifted summing out
  - Sum out representative instance as in propositional variable elimination
  - Exponentiate result for indistinguishable instances

- Correctness: Equivalent ground operation
  - Each instance is summed out
  - Result: factor $f$ that is identical for all instance
  - Multiplying indistinguishable results $\rightarrow$ exponentiation of one representative $f$

[Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a]
QA: LVE in Detail

• E.g., marginal
  • $P(\text{Travel(eve)})$
  • Split atoms $R(..., X, ...) \text{ w.r.t. eve if eve in } \text{dom}(X)$
QA: LVE in Detail

- E.g., marginal
  - \( P(\text{Travel}(\text{eve})) \)
  - Split atoms \( R(\ldots, X, \ldots) \) w.r.t. eve if eve in \( \text{dom}(X) \)
QA: LVE in Detail

- E.g., marginal
- $P(\text{Travel}(\text{eve}))$
- Split atoms $R(..., X, ...)$ w.r.t. $\text{eve}$ if $\text{eve}$ in $\text{dom}(X)$
E.g., marginal
- $P(Travel(eve))$
- Split atoms $R(\ldots, X, \ldots)$ w.r.t. $eve$ if $eve$ in $dom(X)$
- Eliminate all non-query variables
QA: LVE in Detail

- Eliminate $\text{Treat}(X, M)$
QA: LVE in Detail

- Eliminate $\text{Treat}(X, M)$
  - Appears in only one $g$: $g_3$
  - Contains all logical variables of $g_3$: $X, M$
  - For each $X$ constant: the same number of $M$ constants
QA: LVE in Detail

• Eliminate $Treat(X, M)$
  • Appears in only one $g$: $g_3$
  • Contains all logical variables of $g_3$: $X, M$
  • For each $X$ constant: the same number of $M$ constants

✓ Preconditions of lifted summing out fulfilled, lifted summing out possible

\[ X \in \{alice, bob\} \]

\[ \text{Treat}(X, M) \]
LVE in Detail: Lifted Summing Out

- Eliminate \( \text{Treat}(X, M) \) by lifted summing out
  1. Sum out representative
  2. Exponentiate for indistinguishable objects

\[
\left( \sum_{t \in \text{Ter}(\text{Treat}(X,M))} g_3(\text{Epid} = e, \text{Sick}(X) = s, \text{Treat}(X, M) = t) \right)^{\#M|X}
\]
LVE in Detail: Lifted Summing Out

\[
\sum_{t \in r(Treat(X,M))} g_3(\text{Epid} = e, \text{Sick}(X) = s, \text{Treat}(X,M) = t)^{\#M|X}
\]

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LVE in Detail: Lifted Summing Out

\[
\left( \sum_{t \in \mathcal{r}(Treat(X,M))} g_3(\text{Epid} = e, \text{Sick}(X) = s, \text{Treat}(X,M) = t) \right)^{\#M|X}
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LVE in Detail: Lifted Summing Out

\[
\left( \sum_{t \in \{\text{Treat}(X,M)\}} g_3(\text{Epid} = e, \text{Sick}(X) = s, \text{Treat}(X,M) = t) \right)^{#M|X}
\]

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\[
\begin{align*}
\text{Epid} & \quad \text{Sick}(X) & \quad \Sigma \\
\text{false} & \quad \text{false} & \quad 10 \\
\text{false} & \quad \text{true} & \quad 9 \\
\text{true} & \quad \text{false} & \quad 12 \\
\text{true} & \quad \text{true} & \quad 12
\end{align*}
\]

\[
\begin{align*}
\text{Epid} & \quad \text{Sick}(X) & \quad ^\wedge \\
\text{false} & \quad \text{false} & \quad 10^2 \\
\text{false} & \quad \text{true} & \quad 9^2 \\
\text{true} & \quad \text{false} & \quad 12^2 \\
\text{true} & \quad \text{true} & \quad 12^2
\end{align*}
\]
LVE in Detail: Lifted Summing Out

- Result after summing out $\text{Treat}(X, M)$:

$$
\sum_{t \in \text{Treat}(X, M)} g_3(\text{Epid} = e, \text{Sick}(X) = s, \text{Treat}(X, M) = t)^{#M|X}
$$

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$$X \in \{\text{alice, bob}\}$$
LVE in Detail: Lifted Summing Out

- Result after summing out $Treat(X, M)$:

\[
\sum_{t \in \tau(Treat(X,M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t)
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Only here, domain size comes into play → no change in graph / parfactor if domain size changes
Tractability

• Given a model that allows for lifted calculations
  • I.e., no groundings during solving an instance of the problem
• Solving an instance of the problem is possible in time polynomial in domain sizes
  → The query answering algorithm is domain-lifted
• An query answering problem is tractable
  • when it is solved by an efficient algorithm, running in time polynomial in the number of random variables
• Assume that the number of random variables is characterised by domain sizes
  • Then, solving a query answering problem is tractable under domain-liftability
    • Runtime might still be exponential in other terms
    • More general results by Niepert & Van den Broeck (2014)
Indistinguishable Evidence and Query Terms

Evidence

- Observations for instances of a PRV
  - One of the range values
  - Not available
- Treat as groups per observation
  - Shatter model on the groups
- Example: 10 instances observed true

<table>
<thead>
<tr>
<th>( Sick(X^T) )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>0</td>
</tr>
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\[ \text{dom}(X^T) = \{x_1, \ldots, x_{10}\} \]
\[ \text{dom}(X) = \{x_{11}, \ldots, x_n\} \]
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Query Terms
• Indistinguishable instances in query:
  • \( P(\text{Sick}(alice), \text{Sick}(eve), \text{Sick}(bob)) \)
  • Result will have local symmetries, e.g., 2 false and 1 true maps to potential of 2
• Parameterised query: \( P(\text{Sick}(X)) \)
• Use standard LVE
  • Count conversion yields wanted result

<table>
<thead>
<tr>
<th>#_X[\text{Sick}(X)]</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,3]</td>
<td>1</td>
</tr>
<tr>
<td>[1,2]</td>
<td>2</td>
</tr>
<tr>
<td>[2,1]</td>
<td>3</td>
</tr>
<tr>
<td>[3,0]</td>
<td>4</td>
</tr>
</tbody>
</table>
Keeping Indistinguishability over Time

The Power of Indistinguishability
• Marginal distribution queries: $P(A^i_\pi | E_{0:t})$
  • Hindsight: $\pi < t$ (Was there an epidemic $t - \pi$ days ago?)
  • Filtering: $\pi = t$ (Is there currently an epidemic?)
  • Prediction: $\pi > t$ (Will there be an epidemic in $\pi - t$ days?)
• Assignment queries on temporal sequence
Reasoning over Time: Interfaces

- Main idea: Use temporal conditional independences for efficient temporal QA
  - Normally only a subset of random variables influence next time step → interface variables
  - State description of interface from time slice $t-1$ suffices to perform inference on time slice $t$
    → Makes present independent from past / future

Algorithms:
- Propositional: Interface Algorithm [Murphy, 2002]
- Lifted: Lifted Dynamic Junction Tree Algorithm [Gehrke et al, 2018]
Taming Reasoning

- Evidence can ground a model over time
- Non-symmetric evidence
  - Observe evidence for some instances in one time step
  - Observe evidence for a subset of these instances in another time step
  - Split the logical variable slowly over time

Interface carries over splits, leading to slowly grounding a model over time
Undoing Splits

• Need to undo splits to keep reasoning polynomial w.r.t. domain sizes

• Where can splits be undone efficiently?
  • When moving from one time step to the next, i.e., in the interface

• How to undo splits?
  • Find approximate symmetries
  • Merge based on groundings

• Is it reasonable to undo splits?
  • Effect of slight differences in evidence?
  • Impact of evidence vs. temporal model
Is It Reasonable to Undo Splits?

- Approximate forward message
- For each time step the temporal behaviour is multiplied on the forward message
- Indefinitely bounded error due to temporal behaviour
Results

- DBSCAN for Clustering
- ANOVA for checking fitness of clusters
- Right: runtimes
- Below: approximation error

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>Max</th>
<th>Min</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0001537746121</td>
<td>0.0000000001720</td>
<td>0.0000191206488</td>
</tr>
<tr>
<td>2</td>
<td>0.00000000851654</td>
<td>0.0000000000001</td>
<td>0.00000000111949</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000478</td>
<td>0</td>
<td>0.0000000000068</td>
</tr>
</tbody>
</table>
Indistinguishability in Decision Making

The Power of Indistinguishability
Indistinguishability for Decision Making

• Online decision making: Graphical models extended by decision and utility nodes
  • Parameterise decisions to make decisions for whole groups of indistinguishable instances: \( \text{Treat}(X, M) \) (box in graph)
  • PRVs in utility functions to denote identical share in contributed utility \( U \) (diamond in graph): \( \phi_U(\text{Epid}, \text{ Sick}(X)) \)
  • (Dynamic) decision parfactor models, Markov logic decision networks
Indistinguishability for Decision Making

- Inference task: maximum expected utility (MEU) query
  - *Which actions can be expected to lead to the maximum utility?*
  - Standard inference algorithms more or less still work
    - Iterate through all possible decisions, set decisions as evidence, calculate expected utility, store current maximum
    - Solve an MAP query with decision variables as query terms and the other variables in the model to eliminate

Assign same action to group of indistinguishable instances
- Fewer possible decisions to consider \(\rightarrow\) *tractability*!
Indistinguishability for Decision Making

- Offline decision making: solve a (partially observable) Markov decision problem (POMDP)
  - First-order / relational MDPs: indistinguishability in the environment
    [Sanner & Kersting 2012]
  - Based on situation calculus: work with representatives
    - E.g., it is important that a box with medical supplies arrives at a destination but not which one it is in particular (of a set of boxes with medical supplies)
  - Novel propositional situations worth exploring may be instances of a well-known context in the relational setting → exploitation promising
    - E.g., household robot learning water-taps
    - Having opened one or two water-taps in a kitchen, one can expect other water-taps in kitchens to work similarly
      ⇒ Priority for exploring water-taps in kitchens in general reduced
      ⇒ Information gathered likely to carry over to water-taps in other places
- Hard to model in propositional setting: each water-tap is novel
Indistinguishability for Decision Making

• Multi-agent setting: decentralised POMDP [Oliehoek & Amato 2016]
  • Set of agents with
    • Own set of available actions, observations
    • Shared state and reward
• Lifting for agents [B et al. 2022]
  • Agents with indistinguishable behaviour $\rightarrow$ types
    • The same sets of actions, observations available
    • Same strategy / program applies if certain independences hold
  • Groups by types can be treated by representatives
  • Reduce exponential dependence on agent numbers
  • Application: Nanoagent network
Agenda

• Statistical Relational Artificial Intelligence
  • Probabilistic relational models
  • Grounding semantics
  • Context
• The Power of Indistinguishability
  • Lifted query answering and tractability
  • Keeping indistinguishability over time
  • Indistinguishability in decision making

• Summary
The Finish Line: The Power of Indistinguishability

• Lifted query answering and tractability
  • Use information about indistinguishability to speed up inference
  • Tractability in terms of domain sizes through lifting
  • Handle evidence in groups of indistinguishable observations
  • Count values in histograms for lifted queries

• Keeping indistinguishability over time
  • Merge parfactors with bounded error

• Indistinguishability in decision making
  • Relational environment encoded
  • Agent types
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What else is there to do? – Oh, so much…
- Approximating symmetries
- Generalising lifting operators
- More robust learning algorithms
- Privacy
- Ethical behaviour
- Explainability
- …
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Thank you!
Bibliography & Further Papers
Ordered topic-wise and then alphabetically
Bibliography – General


• [De Salvo Braz et al. 05] Rodrigo de Salvo Braz, Eyal Amir, and Dan Roth. Lifted First-order Probabilistic Inference. IJCAI-05 Proceedings of the 19th International Joint Conference on Artificial Intelligence, 2005

• [De Salvo Braz et al. 06] Rodrigo de Salvo Braz, Eyal Amir, and Dan Roth. MPE and Partial Inversion in Lifted Probabilistic Variable Elimination. AAAI-06 Proceedings of the 21st Conference on Artificial Intelligence, 2006


• [Poole 03] David Poole. First-order probabilistic inference. IJCAI 2003: 985-991

Bibliography

Bibliography

- [Braun & Möller 16]

- [Braun & Möller 17]

- [Braun & Möller 17a]
  Tanya Braun and Ralf Möller. Counting and Conjunctive Queries in the Lifted Junction Tree Algorithm. In Postproceedings of the 5th International Workshop on Graph Structures for Knowledge Representation and Reasoning, 2017

- [Braun & Möller 18]

- [Braun & Möller 18a]
  Tanya Braun and Ralf Möller. Parameterised Queries and Lifted Query Answering. In IJCAI-18 Proceedings of the 27th International Joint Conference on Artificial Intelligence, 2018

- [Braun & Möller 18b]
  Tanya Braun and Ralf Möller. Lifted Most Probable Explanation. In Proceedings of the International Conference on Conceptual Structures, 2018

- [Braun & Möller 18c]
  Tanya Braun and Ralf Möller. Fusing First-order Knowledge Compilation and the Lifted Junction Tree Algorithm. In Proceedings of KI 2018: Advances in Artificial Intelligence, 2018

- [Braun & Möller 19]
Bibliography

- [Gehrke et al. 18]

- [Gehrke et al. 18b]

- [Gehrke et al. 18c]

- [Gehrke et al. 19]

- [Gehrke et al. 19b]
  Marcel Gehrke, Tanya Braun, Ralf Möller, Alexander Waschkau, Christoph Strumann, and Jost Steinhäuser. Lifted Maximum Expected Utility. In Artificial Intelligence in Health, 2019

- [Gehrke et al. 19c]

- [Gehrke et al. 19d]

- [Gehrke et al. 19e]

- [Gehrke et al. 19f]
Bibliography – Query Answering

• Ahmadi et al. (2013)

• B (2020)

• B & Möller (2018)

• B & Möller (2019)
Bibliography

• Jaeger & Schulte (2018)

• Kersting et al. (2009)

• Lauritzen & Spiegelhalter (1988)

• Mittal et al. (2019)
Bibliography

• Niepert & Van den Broeck (2014)

• Pearl (1982)

• Poole (2003)

• Poole et al. (2014)
Bibliography

- Singla & Domingos (2008)

- Taghipour et al. (2013)

- Taghipour et al. (2013a)

- Van den Broeck (2011)
Bibliography

• Van den Broeck & Darwiche (2013)

• Van den Broeck & Davis (2012)

• Van den Broeck & Niepert (2015)
Bibliography – Temporal Models

• Ahmadi et al. (2013)

• Gehrke et al. (2018)

• Gehrke et al. (2019)

• Gehrke et al. (2019a)
Bibliography

• Gehrke et al. (2020)

• Mladenov et al. (2017)

• Murphy (2002)

• Venugopal & Gogate (2014)
Bibliography – Decision Making

• B et al. (2022)

• Gehrke et al. (2019b)

• Gehrke et al. (2019c)

• Nath & Domingos (2009)
Bibliography – Decision Making

• Oliehoek & Amato (2016)
  Frans A. Oliehoek and Christopher Amato. A Concise Introduction to Decentralised POMDPs, 2019.

• Sanner & Kersting (2010)