# Learning Argumentation Frameworks SUM 2022 

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18 October 2022

## Outline

1. Context (Explainable AI, Argumentation)
2. Part I: Learning Abstract Argumentation Frameworks
3. Part II: Learning Assumption-Based Argumentation Frameworks
4. Future work

## Context: eXplainable AI (XAI)

Map of Explainability Approaches


From "Principles and Practice of Explainable ML"; Belle\&Papantonis 2021

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- "The majority of what might look like causal attributions turn out to look like argumentative claim-backings" [from Explaining in conversation: Towards an argument model. Antaki,Leudar. Journal of Social Psychology 1992]


## Context: Argumentation



Figure: Argumentation for KR

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- Various argumentation frameworks, e.g. Abstract Argumentation (AA) and Assumption-Based Argumentation (ABA), with lots of applications

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- Can these argumentation frameworks be learnt?

In this talk I will present two approaches to learn AA and ABA frameworks from "examples"

## Argumentation：An illustration

## Am I eligible to claim for UK \＆European Breakdown \＆Recovery Assistance？

You need to think about whether the insurance meets your needs and whether you can claim when you need to．

## You are covered fos

## You are not covered for：

UK and European Breakdown Assistance for account hoider（s）in any private car that they ale
traveling in
$\checkmark$ Anyone driving a private car registered to th account holder and which is being used with ins． her permission．Where the account is in joint names then up to 2 private cars can be covered
$\checkmark$ Assistance provided at home and on the roadside with national recovery and onward travel
$\checkmark$ No call out limit
$\checkmark$ No excess payable
－The cost of replacement parts and associated labour to renair the wabicle
－Prinate cars not registered to the account hoider（s）unless． the account hoider（s）are in the vehicle at the time of
the breakdown
－Motorcydes，motorhomes，caravanettes，commercial vehicles （all types），vars，pick up truds and vehicles being used for hire and reward purposes（such as taxis）
－Vehicles that do not have a valid MOT or are not serviced or maintained in line with manufacturer guidelines
－Vehicles that are more than 7 metres in length， 2.3 metres wide， 3 metres high and weigh more than 3.5 tonnes when fully loaded

## Argumentation: An illustration

COVERED FOR: UK/EU Breakdown Assistance for account holder(s) in any private car they are travelling in
NOT COVERED FOR: private cars not registered to the account holder(s) unless in the vehicle at the time of the breakdown
Mary: account holder traveling in friend's car; car breaks down. Is Mary covered?

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$c$ (mary) is (dialectically) "good"/"strong'" and Mary is covered


## Part I: Learning Abstract Argumentation Frameworks

1. Background (AA frameworks)
2. Problem
3. Solution

Bibliography

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- Cyras, Satoh, Toni: Abstract Argumentation for Case-Based Reasoning. KR2016
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## Background: AA frameworks

$\langle A r g s, A t t\rangle$ where

- Args is a set (the arguments)
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Grounded extension

- Let $G_{0}$ be the set of unattacked arguments in Args.
- For each $i \in \mathbb{N}$, let $G_{i+1} \subseteq$ Args be the set of arguments that $G_{i}$ defends (by attacking all arguments attacking $G_{i}$ ).
Then $G=\cup_{i \in \mathbb{N}} G_{i}$ is the grounded extension of $\langle$ Args, Att $\rangle$.


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$$
\{c(\text { mary }), \text { in(mary })\} \text { is grounded, }\{i n(\text { mary })\} \text { is not }
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The park council deliberates about the interpretation of this rule. So far, it has decided the following:

1. When a young man complained that he was not allowed to use a bicycle in the park, the council decided in his favour.
2. In a similar situation, but regarding a motorized bicycle, the council rejected the complaint.
3. When an ambulance entered the park to rescue an elderly person who was feeling sick, it was considered acceptable.
4. When an unspecified vehicle entered the park to rescue a sick person, this was also considered acceptable.

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How will the park council decide on a new case?
(How can we use the past cases to reason about a new one?)

## Problem: An example (thanks to Guilherme Paulino-Passos)

- Past cases:

1. young man's bicycle
2. motorized bicycle
3. ambulance to rescue sick elderly person
4. unspecified vehicle to rescue sick person

- New case:
- a pickup truck enters the park in order to rescue a sick person

How will the park council decide on this new case?
(How can we use the past cases to reason about a new one?)

## Problem

Given

- a dataset $D$ of cases of the form $(S, o)$
( $S$ features, $o \in\{+,-\}$ outcome)
e.g. $D=\{(\{$ health_emergency, motor, ambulance $\},+)$, ( $\{$ bicycle, motor $\},-$ ) \}
- $D$ is consistent iff there is no $S$ such that $(S,+),(S,-) \in D$. Suppose $D$ is consistent.
- a default outcome $d \in\{+,-\}$
e.g. $d=-$


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\text { e.g. } d=-
$$

Determine/Explain

- the outcome of a focus case (with features) $N$ e.g. $N=\{$ health_emergency, motor $\}$


## Solution: AA-CBR (Example)

Given

- $D=\{(\{b\},+),(\{h\},+),(\{b, m\},-),(\{h, m, a\},+)\}$ (note: $D$ is consistent)
- default outcome: -
- $N=\{h, m\}$


## Solution: AA-CBR (Example)

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- $D=\{(\{b\},+),(\{h\},+),(\{b, m\},-),(\{h, m, a\},+)\}$ (note: $D$ is consistent)
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The grounded extension is $G=\{(\{h, m\}, ?),(\{h\},+)\}$. As $(\},-) \notin G$, the AA-CBR outcome of $(\{h, m\}, ?)$ is + .

## General definition: AA Framework

Let Args $=D \cup\{(N, ?)\} \cup\{(\{ \}, d)\}$.

- for $\left(X, o_{X}\right),\left(Y, o_{Y}\right) \in D \cup\{(\{ \}, d)\},\left(X, o_{X}\right) \operatorname{Att}\left(Y, o_{Y}\right)$ iff

1. $o_{X} \neq o_{Y}$, and
2. $Y \subset X$, and
3. $\nexists\left(Z, o_{X}\right)$ with $Y \subset Z \subset X$
(different outcomes) (specificity) (concision)

- for $\left(Y, o_{Y}\right) \in D,(N, ?) \operatorname{Att}\left(Y, o_{Y}\right)$ iff $Y \not \subset N$
(irrelevance)
e.g. ( $\{$ bicycle $\},+$ ) attacks ( $\},-$ ),
(\{health_emergency, motor\}, ?) attacks
( $\{$ health_emergency, motor, ambulance $\},+$ )


## General definition: Outcomes

We denote the opposite of an outcome $o \in\{+,-\}$ as $\bar{o}$, in the intuitive way:

- $\bar{o}=-$, if $o=+$
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We say that the outcome for the new case $N$ is:
$\checkmark d$, if $(\}, d)$ is in the grounded extension $G$,

- $\bar{d}, \quad$ otherwise


## General definition: Properties

Definition (Nearest cases)
For a case base $C B$ and a new case $N$, a past case $\left(X, o_{X}\right) \in C B$ is nearest to $N$ if $X \subseteq N$, and there is no $\left(Y, o_{Y}\right) \in C B$ such that $Y \subseteq N$ and $X \subset Y$.

Theorem
$G$ contains all the nearest past cases to $N$.
Theorem (Unique past case)
If there is a unique nearest case $(X, o)$ to $N$, then the $A A$ outcome of $N$ is o.

## Explanations in AA-CBR

- Return nearest cases
- typical way in CBR
- shows conflicting evidence in past cases
- Can we do better?
- Idea: use dispute trees


## Dispute trees - default outcome



## Dispute trees - non-default outcome



## Learning AA frameworks: Beyond AA-CBR



- Tabular data (discrete)
- Unstructured data (sentiment analysis)


## Part II: Learning ABA Frameworks

1. Background (ABA frameworks and Logic Programming)
2. Problem
3. Solution

## Bibliography

- Bondarenko, Dung, Kowalski, Toni: An Abstract, Argumentation-Theoretic Approach to Default Reasoning. AIJ 1997
- Proietti, Toni: Learning Assumption-based Argumentation Frameworks. IJCLR 2022


## Background: ABA Frameworks

An $A B A$ framework is a tuple $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$ where

- $\langle\mathcal{L}, \mathcal{R}\rangle$ is a deductive system, with $\mathcal{L}$ a language and $\mathcal{R}$ a set of (inference) rules of the form $s_{0} \leftarrow s_{1}, \ldots, s_{m}$ $\left(m \geq 0, s_{i} \in \mathcal{L}\right.$, for $\left.1 \leq i \leq m\right)$;
- $\mathcal{A} \subseteq \mathcal{L}$ is a (non-empty) set of assumptions;
- — is a total mapping from $\mathcal{A}$ into $\mathcal{L}$, where $\bar{a}$ is the contrary of $a$, for $a \in \mathcal{A}$.


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Example (using schemata)
- $\mathcal{L}=\{p(X), q(X), r(X), a(X), b(X) \mid X \in\{1,2\}\} ;$

$$
\mathcal{R}=\{p(X) \leftarrow a(X), \quad q(X) \leftarrow b(X), \quad r(1) \leftarrow \text { true }\} ;
$$

- $\mathcal{A}=\{a(X), b(X)\}$;
- $\overline{a(X)}=q(X), \quad \overline{b(X)}=r(X)$.


## Background: Logic programming

Flat ABA frameworks $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$ where $\mathcal{L}$ is a set of atoms amount to (normal) logic programs.

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Example (as logic program)
$p(X) \leftarrow \operatorname{not} q(X)$
$q(X) \leftarrow \operatorname{not} r(X)$
$r(1) \leftarrow$

## Background: (flat) ABA/Logic programming semantics

- ABA:
- arguments are deductions of claims using rules and supported by assumptions,
- attacks are directed at the assumptions in the support of arguments;
- Abstract Argumentation-style extension-based semantics


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- $\} \vdash r(1)$ attacks $\{b(1)\} \vdash q(1)$,
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Various notions of "acceptable" extensions (sets of arguments)

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Various notions of "acceptable" extensions (sets of arguments)

One-to-one correspondence between models of logic programs and acceptable extensions in flat ABA
e.g. well-founded model $\sim$ grounded extension

## Problem: An example (Dimopoulos-Kakas 1995)

- Given

1) Background knowledge (ABA framework):

$$
\begin{aligned}
\mathcal{R}=\{ & \operatorname{bird}(X) \leftarrow \text { penguin }(X) \\
& \text { penguin }(X) \leftarrow \operatorname{superpenguin}(X) \\
& \operatorname{bird}(a) \leftarrow, \operatorname{bird}(b) \leftarrow \\
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3) Negative Examples: $\{$ flies(c), flies(d) $\}$

- Determine an ABA framework "generalising" the examples

$$
\begin{aligned}
\mathcal{R}^{\prime}=\{ & \left\{\text { flies }(X) \leftarrow \operatorname{bird}(X), \alpha_{1}(X),\right. \\
& c-\alpha_{1}(X) \leftarrow \operatorname{penguin}(X), \alpha_{2}(X) \\
& \left.c-\alpha_{2}(X) \leftarrow \operatorname{superpenguin}(X)\right\} \cup \mathcal{R} \\
\mathcal{A}^{\prime}=\{ & \left.\alpha_{1}(X), \alpha_{2}(X)\right\} \quad \text { with }{\overline{\alpha_{i}(X)}}^{\prime}=c-\alpha_{i}(X)
\end{aligned}
$$

## Problem: Formally

- $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \vDash s$ indicates that $s \in \mathcal{L}$ is the claim of an argument accepted in all or some (stable, grounded, ...) extensions of $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$.


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is covered by $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$ iff $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \models e$ and is not covered by $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$ iff $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \not \vDash e$.


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- Given background knowledge $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$, positive examples $\mathcal{E}^{+}$and negative examples $\mathcal{E}^{-}\left(\mathcal{E}^{+} \cap \mathcal{E}^{-}=\emptyset\right)$, the goal of ABA learning is to construct $\left\langle\mathcal{L}^{\prime}, \mathcal{R}^{\prime}, \mathcal{A}^{\prime},{ }^{\prime}\right\rangle$ with $\mathcal{R} \subseteq \mathcal{R}^{\prime}, \mathcal{A} \subseteq \mathcal{A}^{\prime}$ and $\forall \alpha \in \mathcal{A}, \bar{\alpha}^{\prime}=\bar{\alpha}$, such that:


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- $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \vDash s$ indicates that $s \in \mathcal{L}$ is the claim of an argument accepted in all or some (stable, grounded, ...) extensions of $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$.
- An example e is covered by $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$ iff $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \models e$ and is not covered by $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle$ iff $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \not \vDash e$.
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- (Existence) $\left\langle\mathcal{L}^{\prime}, \mathcal{R}^{\prime}, \mathcal{A}^{\prime},{ }^{\prime}\right\rangle$ admits at least one extension (under the chosen ABA semantics),


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Solution: Transformation rules for (flat) ABA frameworks

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- Rote Learning. Given atom $p(t)$, add $\rho: p(X) \leftarrow X=t$ to $\mathcal{R}$. Thus, $\mathcal{R}^{\prime}=\mathcal{R} \cup\{\rho\}$.
- Assumption Introduction. Replace $\rho_{1}: H \leftarrow E q s, B$ in $\mathcal{R}$ by $\rho_{2}: H \leftarrow E q s, B, \alpha(X)$ where variables in $X$ are taken from $\operatorname{vars}(H) \cup \operatorname{vars}(B)$ and $\alpha(X)$ is a (possibly new) assumption with contrary $\chi(X)$. Thus,
- $\mathcal{R}^{\prime}=\left(\mathcal{R} \backslash\left\{\rho_{1}\right\}\right) \cup\left\{\rho_{2}\right\}$,
- $\mathcal{A}^{\prime}=\mathcal{A} \cup\{\alpha(X)\}$,
- $\overline{\alpha(X)}^{\prime}=\chi(X)$, and $\bar{\beta}^{\prime}=\bar{\beta}$ for all $\beta \in \mathcal{A}$.


## Solution: Requirements

- Let $\left\langle\mathcal{L}^{\prime}, \mathcal{R}^{\prime}, \mathcal{A}^{\prime},{ }^{\prime}\right\rangle$ be obtained by applying any of Folding, Equality Removal and Subsumption to $\langle\mathcal{L}, \mathcal{R}, \mathcal{A}, 一\rangle$ to modify rules with $p$ in the head. If $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \vDash p(t)$ then $\left\langle\mathcal{L}^{\prime}, \mathcal{R}^{\prime}, \mathcal{A}^{\prime},-\prime\right\rangle \models p(t)$.


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- Let $p\left(t_{1}\right), p\left(t_{2}\right)$ be atoms such that $p\left(t_{1}\right) \neq p\left(t_{2}\right)$ and $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \vDash p\left(t_{1}\right)$ and $\langle\mathcal{L}, \mathcal{R}, \mathcal{A},-\rangle \models p\left(t_{2}\right)$.


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## An illustration

```
Given \(\mathcal{R}=\{\operatorname{step}(1,2) \leftarrow, \operatorname{step}(1,3) \leftarrow, \operatorname{step}(2,4) \leftarrow, \operatorname{step}(2,5) \leftarrow\),
    \(\operatorname{step}(4,6) \leftarrow, \operatorname{step}(5,2) \leftarrow, \operatorname{busy}(3) \leftarrow, \operatorname{busy}(6) \leftarrow\}\);
    \(\mathcal{E}^{+}=\{\)free(1), free(2), free(5) \(\}\),
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- By Equality Removal, we get

$$
\begin{equation*}
\operatorname{free}(X) \leftarrow \operatorname{step}(X, Y) \tag{3}
\end{equation*}
$$

## An illustration (Continued)

$$
\begin{aligned}
\text { Given } \mathcal{R}= & \{\operatorname{step}(1,2) \leftarrow, \operatorname{step}(1,3) \leftarrow, \operatorname{step}(2,4) \leftarrow, \operatorname{step}(2,5) \leftarrow, \\
& \operatorname{step}(4,6) \leftarrow, \operatorname{step}(5,2) \leftarrow, \text { busy }(3) \leftarrow, \operatorname{busy}(6) \leftarrow\} ; \\
\mathcal{E}^{+}= & \{\text {free }(1), \text { free }(2), \text { free }(5)\}, \\
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- Rule (3) free $(X) \leftarrow \operatorname{step}(X, Y)$ covers $\mathcal{E}^{+}$as well as free $(4) \in \mathcal{E}^{-}$.


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$$
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\end{aligned}
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$$
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$$

The final learnt set of rules is $\mathcal{R} \cup\{(4),(6)\}$.

## Explanations: Dispute trees?

Mary ( $m$ ): account holder traveling in friend's car (c); car breaks down

$$
\begin{aligned}
\mathcal{R}: \quad & \operatorname{cov}(m, c) \leftarrow \operatorname{ah}(m), \operatorname{tr}(m, c), \operatorname{pr}(c), \operatorname{not} \neg \operatorname{cov}(m, c) \\
& \neg \operatorname{cov}(m, c) \leftarrow \neg \operatorname{reg}(c, m), \operatorname{not} \operatorname{cov}^{\prime}(m, c) \\
& \operatorname{cov}^{\prime}(m, c) \leftarrow \operatorname{in}(m, c) \\
& a h(m) \leftarrow \quad \operatorname{tr}(m, c) \leftarrow \quad \operatorname{pr}(c) \leftarrow \\
& \neg \operatorname{reg}(c, m) \leftarrow \quad \operatorname{in}(m, c) \leftarrow
\end{aligned}
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$\mathcal{L}: \quad$ Herbrand base of $\mathcal{R}$ plus (all) NAF literals
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\begin{array}{llc} 
& \neg \operatorname{cov}(m, c) \leftarrow \neg \operatorname{reg}(c, m), \text { not } \operatorname{cov}^{\prime}(m, c) & \mathrm{P}:\{\operatorname{not} \neg \operatorname{cov}(m, c)\} \vdash \operatorname{cov}(m, c) \\
& \operatorname{cov}(m, c) \leftarrow \operatorname{in}(m, c) & \mid \\
& a h(m) \leftarrow \quad \operatorname{tr}(m, c) \leftarrow \quad \operatorname{pr}(c) \leftarrow & \mathrm{O}:\left\{\operatorname{not} \operatorname{cov}^{\prime}(m, c)\right\} \vdash \neg \operatorname{cov}(m, c) \\
& \neg \operatorname{reg}(c, m) \leftarrow \quad \operatorname{in}(m, c) \leftarrow & \mid \\
\mathcal{L}: & \text { Herbrand base of } \mathcal{R} \text { plus (all) NAF literals } & \mathrm{P}:\{ \} \vdash \operatorname{cov}^{\prime}(m, c)
\end{array}
$$

$\mathcal{A}$ : (all) NAF literals
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## Future Work

- Learning AA frameworks: non-discrete data?


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