Forgetting Formulas and Signature Elements in Epistemic States

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Overview

1 Forgetting of Signature Elements

2 Ranking Functions, Marginalization & Forgetting

3 Forgetting of Formulas

4 Summary

Background

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$\models$	Model relation / logical entailment
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Because the specific signature is very important here, we attach the corresponding signature in the index:

$$\mathcal{L}_\Sigma, Cn_\Sigma, \models_\Sigma, \;\; \mathcal{L}_P, Cn_P, \models_P, \;\; \mathcal{L}_{\Sigma'}, \;\; \ldots$$

where $\Sigma, \Sigma', P, P', \ldots$ Signatures (finite set of prop. variables)

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The operator $\mathcal{F}(\Gamma, P)$ is fully characterized by:

(DFP-1) $\Gamma \models \mathcal{F}(\Gamma, P)$
(DFP-2) If $\Gamma \models \Gamma'$, then $\mathcal{F}(\Gamma, P) \models \mathcal{F}(\Gamma', P)$
(DFP-3) $\mathcal{F}(\Gamma, P) = Cn_{\Sigma \setminus P}(\mathcal{F}(\Gamma, P))$
(DFP-4) If $P' \subseteq P$, then $\mathcal{F}(\Gamma, P) = \mathcal{F}(\mathcal{F}(\Gamma, P'), P)$
(DFP-5) $\mathcal{F}(\Gamma, P \cup P') = \mathcal{F}(\Gamma, P) \cap \mathcal{F}(\Gamma, P')$
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(DFP-7) $\mathcal{F}(\Gamma, P) = Cn_\Sigma(\mathcal{F}(\Gamma, P)) \cap \mathcal{L}_{\Sigma \setminus P}$

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- Forgetting of Formula:
  - Translation of (DFP-1)–(DFP-7) to epistemic states (and forgetting of formulas)
  - Providing a characterization theorem

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Example for $\Sigma = \{a, b\}$

2	$a\overline{b}$
1	$\overline{a}b$, $\overline{a}\overline{b}$
0	ab
	κ^{ab}

$$Bel(\kappa^{ab}) = Cn_\Sigma(a \wedge b)$$

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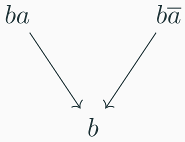

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$$\begin{array}{ccc} ba & & b\overline{a} \\ & \searrow \quad \swarrow & \\ & b & \end{array}$$

Marginalization of κ to Σ' [Künstliche Intelligenz 2019]:

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Theorem

Let $\Gamma \subseteq \mathcal{L}_{\Sigma}$ be a set of formulas and κ an OCF over signature Σ, then

$$\mathcal{F}(Bel(\kappa), P) = Bel(\kappa_{|(\Sigma \setminus P)})$$

holds for each signature $P \subseteq \Sigma$.

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We considered forgetting of *signature elements*:

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Switching to the setting of belief change (Darwiche and Pearl (1997)):

$\mathcal{E} = \{\Psi, \Psi_1, \ldots\}$	Set of epistemic states
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Can we carry over ideas from forgetting of signature elements?

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No addition of beliefs, i.e. $\mathcal{F}(\Gamma, P) \subseteq \Gamma$

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Intermediate steps have no effects

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Note that we have no counter-parts for:

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Additional success condition:

(DFPes-6)$_\mathcal{L}$ If $\phi \not\equiv \top$, then $Bel(\Psi \circ \phi) \not\models \phi$

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Theorem (Triviality Result)

*A belief change operator $\circ$ satisfies **(DFPes-1)$_\mathcal{L}$**-**(DFPes-6)$_\mathcal{L}$** if and only if $Bel(\Psi \circ \phi) \equiv \top$ holds for each $\phi \in \mathcal{L}$.*

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You can do the following:

$$
\begin{aligned}
Bel(\Psi \circ (\phi \wedge \psi)) &\overset{\phi \wedge \psi \models \phi}{=} Bel((\Psi \circ \phi) \circ \phi \wedge \psi) && \textbf{((DFPes-3)}_\mathcal{L}\textbf{)} \\
&= Bel(\Psi \circ (\phi \vee (\phi \wedge \psi))) && \textbf{((DFPes-5)}_\mathcal{L}\textbf{)} \\
&= Bel(\Psi \circ \phi). && \textbf{((SI))}
\end{aligned}
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Future work:

- Representation theorems for subsets and variations of **(DFPes-1)**$_{\mathcal{L}}$–**(DFPes-6)**$_{\mathcal{L}}$
- Connection to AGM contraction