Characterizing Multipreference Closure with System W

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Introduction (1/2)

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$\rightarrow$ Different approaches to draw inferences,
e.g., p-entailment, system Z, ...

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- background on conditionals and preferential models
- recall system W
- recall MP-closure
- provide semantical characterization of MP-closure with system W
  (which is less involved than the original definition/characterization of system W)

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Three valued semantic [deFinetti 1937]:

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Reasoning: Given a belief base, find all conditionals entailed by it.

Preferential Models

Definition (preferential model [Lehmann Magidor 1992])

A triple $\mathcal{M} = \langle S, l, \prec \rangle$ consisting of

- a set S of *states*,
- a function $l : S \rightarrow \Omega$, and
- a strict partial order on S

such that for every $A \in \mathcal{L}_\Sigma$: $\hat{A} = \{s \mid s \in S,\, l(s) \models A\}$ is smooth.

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$\mathcal{M}$ is a model for belief base Δ if $\mathcal{M}$ accepts all conditionals in Δ.

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- take into account which conditionals are falsified by a world
- inference relation based on a partial ordering on worlds

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System W, $\mathrel{|\!\sim}^{\mathsf{w}}_{\Delta}$[Komo Beierle 2022]

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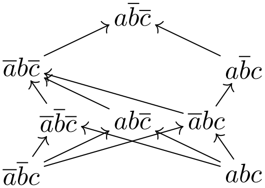

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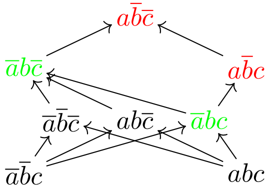

Entailment: e.g. $\overline{a}b \vee a\overline{b} \mid\!\sim^{\mathsf{w}}_{\Delta} \overline{a}b$

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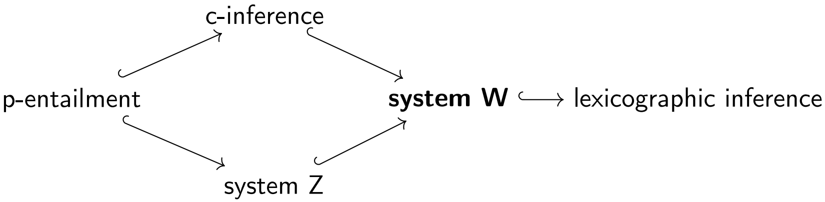

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Definition (system W preferential model $\mathcal{M}^{\mathsf{w}}(\Delta)$)

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Proposition

For consistent Δ*:* $A \mathrel{|\!\sim}_{\mathcal{M}^{\mathsf{w}}(\Delta)} B$ *iff* $A \mathrel{|\!\sim}^{\mathsf{w}}_{\Delta} B$.

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Definition (MP-basis [Giordano Gliozzi 2021])

A set $D \subseteq \Delta$ is an *MP-basis* for A if

- A is consistent with $\tilde{D} = \{B \rightarrow C \mid (C|B) \in D\}$, and
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Definition (MP-closure [Giordano Gliozzi 2021])

$$A \mid\!\sim^{MP}_{\Delta} B$$

if for all MP-bases D of A it holds that $\tilde{D} \cup \{A\} \models B$.

$\rightarrow$ Similar construction as in the definition of lexicographic inference [Lehmann 1995].

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- Step 1: Characterization of MP-closure with *MP-models* [Giordano Gliozzi 2021].
- Step 2: Show system W preferential models are MP-models.
- Step 3: Prove main theorem.

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Proposition (MP-closure representation theorem [GG21])

$$A \mid\!\sim_\Delta^{MP} B \quad \textit{iff} \quad (B|A) \textit{ is accepted by every MP-model of } \Delta$$

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Proposition

The system W preferential model $\mathcal{M}^{\mathsf{w}}(\Delta)$ *is an MP-model of* Δ.

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Proposition ([Giordano Gliozzi 2021])

Let $\mathcal{N}, \mathcal{N}'$ be MP-models of Δ.

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- Further investigate the connections to reasoning with first-order-conditionals
- Generalize system W for belief bases containing strict knowledge
  (i.e. belief bases enforcing impossible worlds)